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## APPLIED PHYSICS REVIEWS

# Constructal law of design and evolution: Physics, biology, technology, and society

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This is a review of the theoretical and applied progress made based on the Constructal law of design and evolution in nature, with emphasis on the last decade. The Constructal law is the law of physics that accounts for the natural tendency of all flow systems (animate and inanimate) to change into configurations that offer progressively greater flow access over time. The progress made with the Constructal law covers the broadest range of science, from heat and fluid flow and geophysics, to animal design, technology evolution, and social organization (economics, government). This review presents the state of this fast growing field, and draws attention to newly opened directions for original research. The Constructal law places the concepts of life, design, and evolution in physics.
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### TABLE OF CONTENTS



### I. CONSTRUCTAL LAW

A law of physics is a concise statement that summarizes a phenomenon that occurs in nature. The Constructal-law field started from the realization that "design" is a universal physics phenomenon. It unites the animate with the inanimate over an extremely broad range of scales, from the tree design of the snowflake, to animal design and the tree design of the Amazon river basin. Figure 1 makes this point with just two examples of the same volume-point flow architecture, one inanimate and the other animate, to which we could have added many more (e.g., lightning, vascularized living tissues, city traffic, the spreading of new ideas on the globe).

The concepts of life, design, and future (evolution) were placed firmly in physics by the Constructal law, stated in 1996:[1,2]

> "For a finite-size flow system to persist in time (to live), its configuration must evolve in such a way that provides greater and greater access to the currents that flow through it."

According to the Constructal law, a live system is one that has two universal characteristics: It flows (i.e., it is a nonequilibrium system in thermodynamics), and it morphs freely toward configurations that allow all its currents to flow more easily over time. Life and evolution are a physics phenomenon, and they belong in physics.[1,3]

The Constructal law is a field that is expanding rapidly in physics, biology, technology, and social sciences. We reviewed the field in 2006,[4] and now the field is expanding even more rapidly. No less than 14 books have been published on the Constructal law since 2006.[5–18] On April 3, 2013, the entry "constructal" on ISI (Institute of Scientific Information) revealed an h index of 39 and a total number of 6915 citations. On Google Scholar, the word "constructal" yielded 2310 articles and books. The objective of the present review is to outline the current state of this fast growing research activity with special focus on the post-2006 literature.

To see the position of design in nature as a universal phenomenon of physics, it is necessary to recall that thermodynamics rests mainly on two laws, which are both "first principles." The first law commands the conservation of energy in any system. The second law commands the presence of irreversibility (i.e., the generation of entropy) in any system: By itself, any stream flows naturally one way,

 

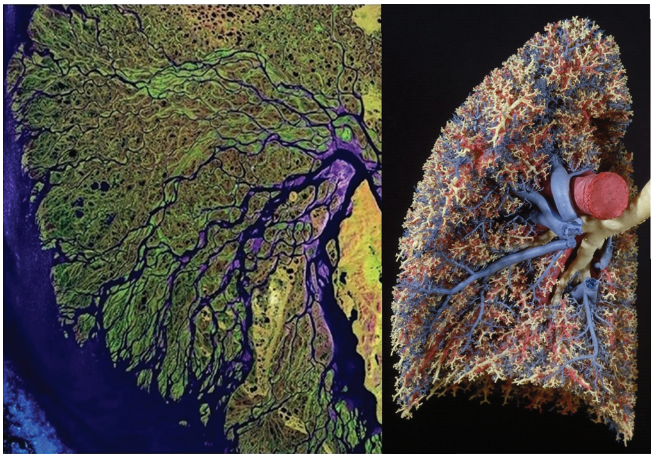

FIG. 1. Design is a universal phenomenon in nature. It is physics. It happens naturally when something is flowing and it is free to morph. Design unites the animate with the inanimate. The left side shows the delta of the Lena River in northern Siberia. The right side shows a cast of the human lung. Courtesy of Ewald Weibel, Professor Emeritus of Anatomy, University of Bern.

from high to low. The permanence and extreme generality of the two laws are consequences of the fact that in thermodynamics the "any system" is a black box. It is a region of space, or a collection of matter without specified shape and structure. The two laws are global statements about the balance or imbalance of the flows (mass, heat, work, entropy) that flow into and out of the black box.

Nature is not made of boxes without configuration (Fig. 1). The systems that we discern in nature have shape and structure. They are macroscopic, finite size, and recognizable as patterns—sharp lines on a diffuse background. They have configurations, maps, rhythms, and sounds. They are simple: Their complexity is modest, because if it were not modest we would not be able to discern them and to question their existence. The very fact that they have names (river basins, blood vessels, trees) indicates that they have unmistakable appearances.

Reference 1 drew attention to the fact that the laws of thermodynamics do not account *completely* for the systems of nature, even though scientists have built thermodynamics into thick books in which the two laws are just the introduction. The body of thermodynamics is devoted to describing, designing, and optimizing things that seem to correspond to systems found in nature, or to devices that can be used by humans to make life easier. Nowhere is this more evident than in engineering, where the method of entropy generation minimization[19,20] is recognized as thermodynamics, even though neither of the two laws accounts for the natural occurrence of "design" or "optimization" phenomena.

If physics is to account for the systems of nature *completely*, then thermodynamics must be strengthened with an additional self-standing law (i.e., with another first principle) that covers all phenomena of design occurrence and evolution. This addition to physics is the Constructal law.[1]

The Constructal law is not a statement of optimization, maximization, minimization, or any other mental image of "end design" or "destiny." The Constructal law is about the direction of evolution in time, and the fact that the design phenomenon is not static: It is dynamic, ever changing, like the images in a movie at the cinema. Evolution never ends. This is important to keep in mind, because there is a growing list of *ad hoc* proposals of optimality (end-design), but each addresses a narrow domain, and, as a consequence, the body of optimality statements that have emerged is self-contradictory, and the claim that each is a general principle is easy to refute:[21]

(i) Minimum entropy generation (production) and maximum efficiency are used commonly in engineering and biology.
(ii) Maximum entropy production (MEP) is being invoked in geophysics.
(iii) Maximum "fitness" and "adaptability" (robustness, resilience) are used in biology.
(iv) Minimum flow resistance (fluid flow, heat transfer, mass transfer) is invoked in engineering, river mechanics, and physiology.
(v) Maximum flow resistance is used regularly in physiology and engineering, e.g., maximum resistance to loss of body heat through animal hair and fur, or through the insulation of power and refrigeration plants, the minimization of fluid leaks through the walls of ducts, etc.
(vi) Minimum travel time is used in urban design, traffic, and transportation.
(vii) Minimum effort and cost is a core idea in social dynamics and animal design.
(viii) Maximum profit and utility is used in economics.
(ix) Maximum territory is used for rationalizing the spreading of living species, deltas in the desert, and empires.
(x) Uniform distribution of maximum stresses is used as an "axiom" in rationalizing the design of botanical trees and animal bones.
(xi) Maximum growth rate of flow disturbances (deformations) is invoked in the study of fluid flow disturbances and turbulence.
(xii) Maximum power was proposed in biology, physics, and engineering.

The optimality statements are contradictory and disunited, yet they demonstrate that the time for placing the "design" phenomena in science is now. The progress made with the Constructal law[21–24] shows that the diversity of phenomena addressed with the *ad hoc* statements (i)–(xii) are manifestations of the single natural tendency that is expressed by this law of physics. For example, the contradiction between (i) minimum and (ii) MEP was resolved based on the Constructal law in 2006:[25] Both (i) and (ii) are covered by the Constructal law.

The reach of the Constructal law was broadened by the concept of the flow of stresses,[9,26] which accounts for the emergence of solid shapes and structures in vegetation, skeleton design, and technology. The flow of stresses is an integral part of the design-generation phenomenon of moving mass more and more easily on the landscape.

The Constructal law can be used to fast-forward design in engineering and social organization, cf. Secs. VIII–XI. This is useful, but the imagined end design (min, max) neither is reachable in nature nor it is to be confused with the phenomenon and the law of physics, which is the natural tendency (the direction in time) that points to it. The time direction is the natural phenomenon, and the law of physics that governs this natural phenomenon is the Constructal law.

## II. TREE-SHAPED DESIGNS: CONDUCTION, FLUID FLOW, AND CONVECTION

The Constructal law statement is general. It does not use words such as *tree, complex* versus *simple,* or *natural* versus *engineered*. There are several classes of flow configurations in nature, and each class can be derived from the Constructal law in several ways: analytically or numerically, approximately or more accurately, blindly (random search) or using strategy (shortcuts), and so on. Classes that our group treated in detail, and by several methods, are the cross-sectional shapes of ducts, the cross-sectional shapes of rivers, internal spacings, and tree-shaped architectures.

Regarding the tree architectures, we treated them not as models but as fundamental problems of access to flow: volume to point, area to point, line to point, and the respective reverse flow directions. Important is the geometric notion that the "volume," the "area," and the "line" represent infinities of points.

The theoretical discovery of trees stems from the decision to connect one point (source, or sink) with an *infinity* of points (volume, area, line). It is the reality of the continuum (the infinity of points) that is routinely discarded by modelers who approximate the space with a finite number of discrete points and then cover the space with drawings made of sticks, which cover the space incompletely (and from this, fractal geometry). The reality of the continuum requires a study of the interstitial spaces between the tree links. The interstices can only be bathed by high-resistivity diffusion (an invisible, disorganized flow), whereas the tree links serve as conduits for low-resistivity organized flow (visible streams, ducts).

The two modes of flowing with imperfection (irreversibility), the interstices and the links, must be *balanced* so that together they ease the global flow. The flow architecture is the graphical expression of the balance between channels and their interstices. The deduced architecture (tree, duct shape, spacing, etc.) is the *distribution* of imperfection over the available flow space. It is the architecture for access into and out of the flow space, which is finite. Those who model natural trees and then draw them as black lines on white paper (while not struggling to discover the layout of every black line on its allocated white patch) miss half of the drawing. The white is as important as the black.

The Constructal-law discovery of tree-shaped flow architectures was based on three approaches. It started in 1996 with an analytical shortcut[1,2] based on several simplifying assumptions: 90 angles between stem and tributaries, a construction sequence in which smaller optimized constructs are retained, constant-thickness branches, and so on. Months later, we solved the same problem numerically[27] by abandoning most of the simplifying assumptions (e.g., the construction sequence) used in the first papers. In 1998, we reconsidered the problem in an area-point flow domain with randomly moving low-resistivity blocks embedded in a high-resistivity background[28,29] with Darcy flow (permeability instead of conductivity and resistivity). Along the way, we found better performance and trees that look more "natural" as we progress in time, that is, as we endowed the flow structure with more freedom to morph.

Figure 2 shows the most recent tree design for conduction in a heat generating medium with high-conductivity channels that are the most free to morph.[30] Darcy fluid flow is one form of "diffusion," i.e., the same physics phenomenon as thermal diffusion (Fourier conduction) and electrical diffusion (Ohm conduction). Yet, it is becoming fashionable to take the original work[28,29] (e.g., Fig. 3), and publish it as new after replacing one diffusion terminology with another.

The constructal literature is expanding rapidly in the domain of tree-shaped designs for conduction,[31–36] fluid flow,[37–45] and convective heat transfer.[46–50] A central feature of these designs is the notion that when channels bifurcate or coalesce their diameters should change by certain factors, so that the overall flow through the architecture is facilitated. The best known design rule of this kind is the Hess-Murray rule ($D_1/D_2 = 2^{1/3}$) for selecting the ratio of channel diameters at a bifurcation.[9] This rule was extended by constructal design in several directions: to junctions with n branches ($D_1/D_2 = n^{1/3}$), to bifurcations with two identical branches (length $L_2$, diameter $D_2$), and one stem ($L_1$, $D_1$) on an area of fixed size ($L_1 \times 2L_2$) in fully developed laminar flow,[9]

$$\frac{D_1}{D_2} = 2^{1/3}, \quad \frac{L_1}{L_2} = 2^{1/3}, \tag{1}$$

and fully developed turbulent flow,[9]

$$\frac{D_1}{D_2} = 2^{3/7}, \quad \frac{L_1}{L_2} = 2^{1/7}, \tag{2}$$

and to bifurcations with unequal branches ($L_2$, $D_2$ and $L_3$, $D_3$) (problem 4.4 in Ref. 9). All these developments come from evolving the flow configuration in accord with the

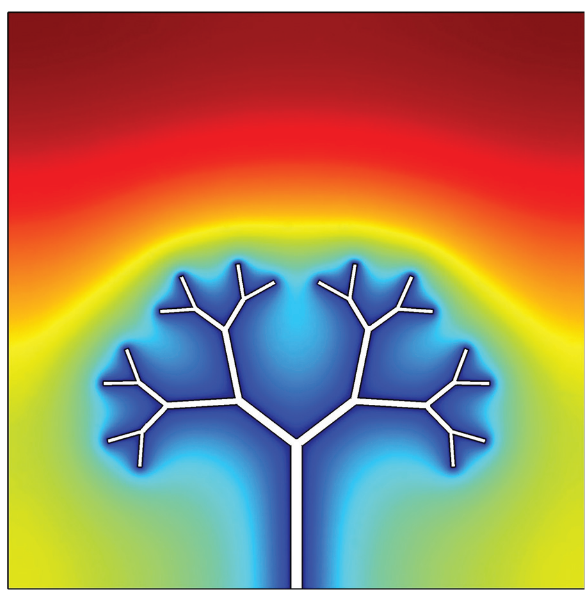

FIG. 2. Constructal invasion of a conducting tree into a conducting body.

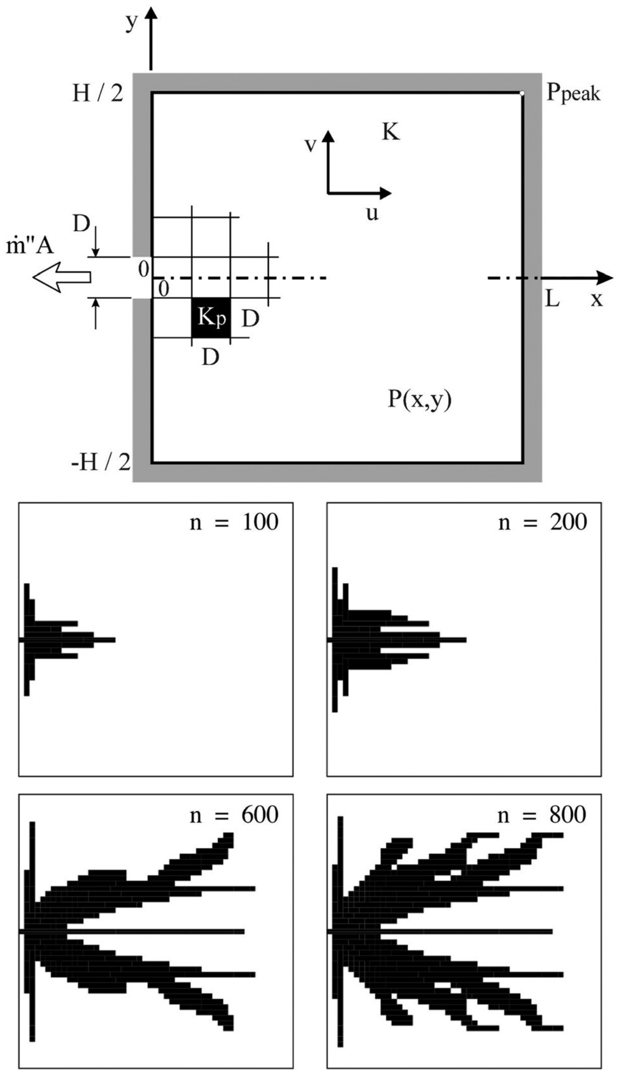


FIG. 3. Darcy flow on a square domain with low permeability (K) and high permeability ($K_p$). In time, K grains are searched and replaced by $K_p$ grains such that the overall area-to-point flow access is increased the fastest. Courtesy of Professor Marcelo R. Errera, Federal University of Parana, Brazil.

Constructal law, toward providing greater access, which led analytically to minimal flow resistance in T- and Y-shaped (and more complicated) constructs of tubes and other channels, as in the trees matched canopy to canopy of Ref. 51.

Here, we point out that the uniform distribution of imperfection in the constructal design [Eqs. (1) and (2)] is the same as the uniform distribution of fluid residence time in the channels. This means that the time ($t_1$) spent by the fluid in the $D_1$ tube is the same as the time ($t_2$) spent in the $D_2$ tube. The residence time in any tube is $t \sim L/U \sim LD^2/\dot{m}$, where U and $\dot{m}$ are the mean fluid speed and mass flow rate, respectively. Next, at a bifurcation we note $\dot{m}_2/\dot{m}_1 = 1/2$, and with the laminar flow architecture of Eq. (1) we obtain

$$\frac{t_1}{t_2} \sim \frac{L_1}{L_2}\frac{\dot{m}_2}{\dot{m}_1}\frac{D_1^2}{D_2^2} = 2^{1/3}\,\frac{1}{2}\,(2^{1/3})^2 = 1. \qquad (3)$$

Similarly, for the turbulent flow design of Eq. (2) we obtain

$$\frac{t_1}{t_2} \sim \frac{L_1}{L_2}\frac{\dot{m}_2}{\dot{m}_1}\frac{D_1^2}{D_2^2} = 2^{1/7}\,\frac{1}{2}\,(2^{3/7})^2 = 1. \qquad (4)$$

Svelteness[9] is a new property of flow architectures, which was brought to light by the constructal design of flow architectures. The svelteness Sv is the ratio of the external length scale of the flow design (for example, $A^{1/2}$, if the area of the flow layout is A) divided by the internal length scale of the flow design (for example, $V^{1/3}$, if the total volume occupied by the flow is V). A flow architecture has three main characteristic: sizes, aspect ratios (shapes), and svelteness, i.e., the relative thinness of the lines of its drawing. Svelteness is intimately tied to the flow performance of the architecture.[9,52]

## III. COMPACT HEAT AND MASS TRANSFER

A major field of applied research for Constructal-design architectures is the development of compact (high density) architectures for heat and mass transfer. This activity began with the discovery of optimal spacings for channels with natural[53] and forced[54] convection (for a review, see Ref. 9), and the development of tree-shaped heat exchangers for high heat transfer density in a confined space.[55] Today, this activity continues on several fronts.

Fins, or extended solid surfaces, were developed most recently in Refs. 56–65. Lorenzini, Biserni, and Rocha[66,67] pioneered the field of "inverted fins," which are cavities with particular architectures (tree-, T-, H-shaped) built into solid walls for the purpose of enhancing the convective thermal contact between the wall and the surrounding fluid flow.

Spacings for natural and forced convection were reported by Bello-Ochende, Meyer and coworkers,[68–71] Canhoto and Reis,[72] Gosselin and coworkers,[73,74] Narasimhan and coworkers,[75,76] Zamfirescu and Dincer,[77] as well as by Refs. 78–83.

The dendritic heat exchanger proposed in Ref. 55 was designed, built, and tested by Raja *et al.*[84] Heat exchanger structures for fuel cells were reported in Refs. 77 and 85–87. Desalination, humidification, and dehumidification applications were studied by Mehrgoo and Amidpour.[88,89] Microreactors were developed by Mathieu-Potvin and Gosselin[90] and Chen *et al.*[91] Microfluidic structures were studied by da Silva and coworkers.[92,93] Constructal design continues to be used in more classical applications such as shell and tube heat exchangers,[94] furnaces,[95] reactors[96–99] and distributors,[100–102] and maximum heat transfer density.[103,104] Boiling in dendritic channels was proposed by Bonjour and coworkers[105,106] and Liu *et al.*[107]

Industrial applications of constructal design were reported for steam generators,[108,109] steam turbines,[110] furnaces for heating streams of solid metal,[111,112] cross-flow heat exchangers,[113] and solar power plants.[114–117] In particular, the configuration of a slender enclosure can be optimized such that the radiation heating of a stream of solid is performed with minimal fuel consumption at the global level. The solid moves longitudinally at constant rate through the enclosure. The enclosure is heated by gas burners distributed arbitrarily, in a manner that is to be determined. The total contact area for heat transfer between the hot enclosure and the cold solid is fixed. The minimal global fuel consumption is achieved when the longitudinal distribution of heaters is nonuniform, with more heaters near the exit than the entrance. The reduction in fuel consumption relative to when the heaters are distributed uniformly is of order 10%. Tapering the plan view (the floor) of the heating area yields an additional reduction in overall fuel consumption. The best shape is when the floor area is a slender triangle on which the cold solid enters by crossing the base, Fig. 4. These architectural features recommend the proposal to organize the flow of the solid as a dendritic design, which enters as several branches and exits as a single hot stream of prescribed temperature.[112]

## IV. VASCULAR DESIGN

A distinct trend in constructal design is the development of vascular flow architectures, which fill bodies (structural members) and endow them with volumetrically distributed functions such as self-healing[118,119] and self-cooling.[120] This work is driven by applications in smart materials, smart structures, the design of future aircraft,[121] and the cooling of progressively more compact electronics.

On the fundamental side, the key question is why should a "vascular" flow architecture emerge in the animal, and in the engineered smart body? "Vascular" means that the stream bathes the entire volume almost uniformly, by flowing as two trees matched canopy to canopy.[122] First, the stream enters the volume by distributing its flow like a river delta. Second, the stream reconstitutes itself and flows out of the volume like a river basin.

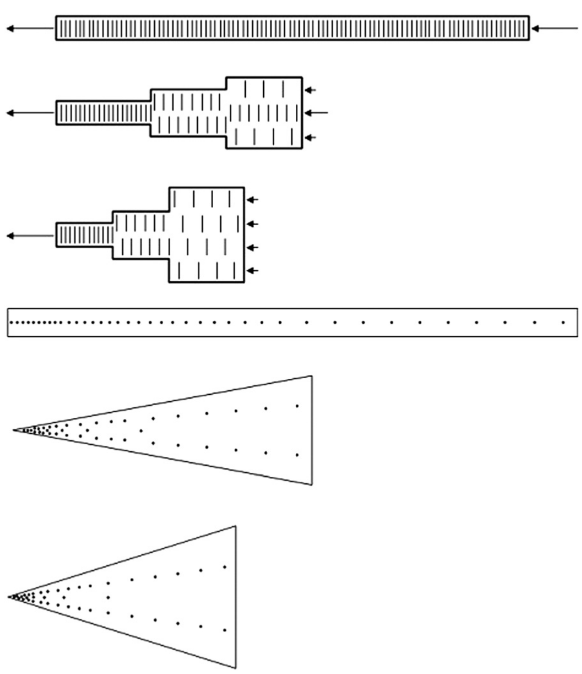

FIG. 4. The distribution of the flow of steel on the furnace floor: uniform density, on a floor with uniform width (top) and increasing density, on triangular floors (middle and bottom). The total flow rate and the floor area are the same in each drawing. The lower three drawings show the distribution of overhead heaters on the area occupied by the upper three designs.[112] Reprinted with permission from J. Appl. Phys. **107**, 114910 (2010). Copyright 2010 American Institute of Physics.

Kim *et al.*[123] showed that tree-tree architectures recommend themselves for all the volumes bathed by single streams in laminar flow, and that in larger volumes each tree must have more levels of branching or coalescence. Cetkin *et al.*[124] demonstrated the same trend for trees with turbulent flow. Figures 5 and 6 summarize these findings, for both laminar and turbulent flow. The size of the volume is represented by the number n, which is the number of elemental volumes bathed by a single capillary stream ($\dot{m}_e$, Fig. 5). The construction steps (1), (2), and (3) of Fig. 5 illustrate how the tree-tree volume acquires a more complex architecture as the number of branching levels increases. The total mass flow rate is fixed. Figure 6 shows how the relative pressure drop $\Delta P/\Delta P_1$ decreases as the size (n) increases. This decrease is possible only if the architecture is free to change abruptly from (1), to (2), and (3). The evolution toward better flow performance in larger systems must be stepwise (revolutionary), not gradual.

The vascular design literature is expanding rapidly, from architectures for self-healing[125,126] to trees matched canopy to canopy in vascular bodies.[127–136] Thermal characteristics and the heat transfer performance of vascular designs are documented in Refs. 137–144. Vascular porous structures were designed for electrokinetic mass transfer in Ref. 145, and for heat transfer in biological tissues in Refs. 146–149. Vascular designs for cooling a plate heated by a randomly moving energy beam were developed by Cetkin *et al.*[150]

## V. THE FLOW OF STRESSES

Design occurs in nature not only in fluid flow systems such as river basins and human lungs (Fig. 1) but also in solid structures such as animal skeletons,[151] vegetation,[26] and bodies of vehicles.[152] Solid structures were brought under the Constructal law by the view that they are bodies shaped for the *flow of stresses*.[9,26] When stresses flow from one end to the other of a structural member without obstacles (strangulations, stress concentrations), the member carries the imposed load with minimum material. The easiest flow of stresses means the lightest and strongest member, and the most efficient animal or vehicle that uses that member as support structure. At bottom, the constructal design of the flow of stresses in solids is a manifestation of the grand constructal design of the flow of mass on the globe.

The flow of stresses as a morphing flow system was proposed[9,26] in order to predict the entire architecture of vegetation, from roots to trunks, canopies, and the floor of the forest. Since then, the flow of stresses has become an integral part of constructal design.[9] Plates can be shaped (tapered) so that stresses flow through them in "boundary layer" fashion.[153] Bars and linkages in compression and buckling can be shaped and sized so that they carry their loads with minimal material. The vascular designs for volumetric cooling (Sec. IV) can be complemented by the shaping and distributing of channels for maximum strength and thermal performance at the same time.[154–156]

## VI. ANIMAL DESIGN AND SPORTS EVOLUTION

The Constructal law and the global design of nature constitute a unified view of evolution. This theoretical view

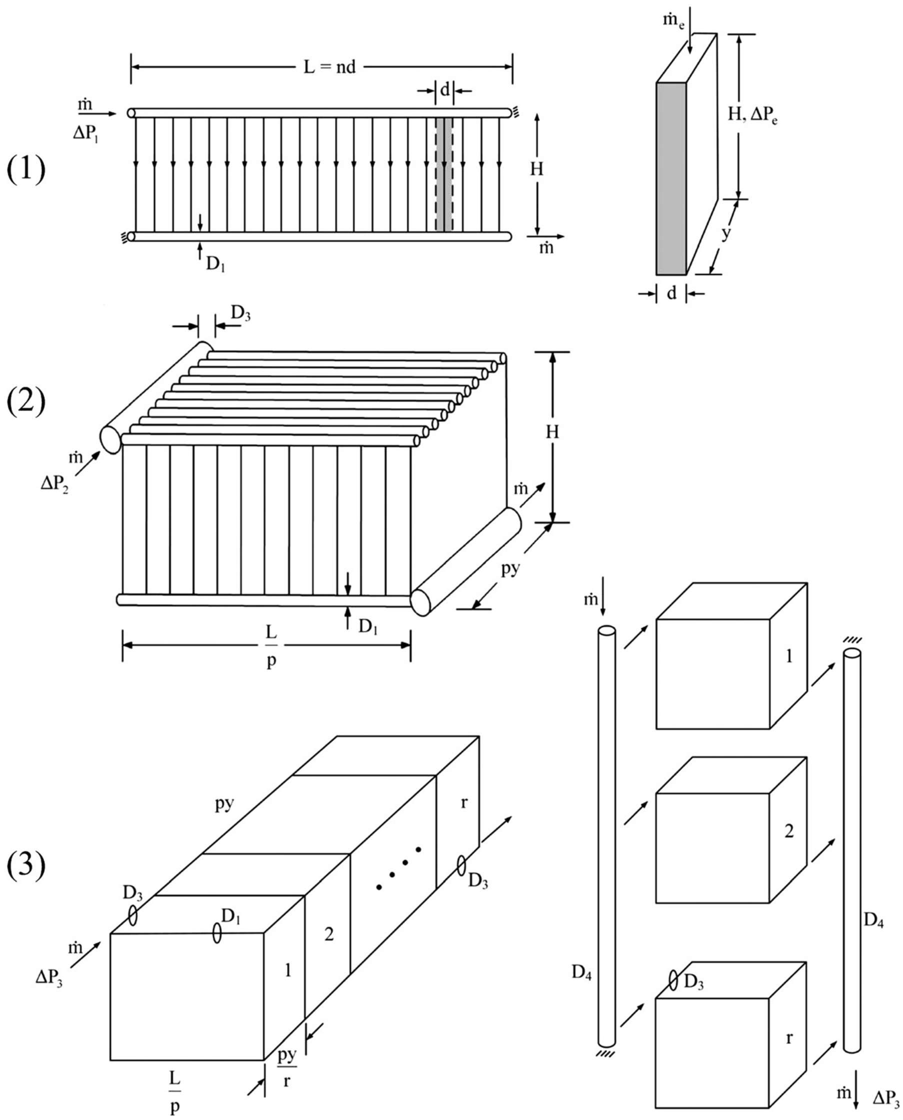


FIG. 5. A volume is bathed by a single stream that flows as two trees matched canopy to canopy: (1) Elemental volumes stacked as a deck of cards; (2) Trees with two branching levels; (3) Trees with three branching levels.[123,124] Reprinted with permission from J. Appl. Phys. **103**, 123511 (2008); **107**, 114901 (2010). Copyright 2010 American Institute of Physics.

predicts evolution in all the diverse domains in which evolutionary phenomena are observed, recorded, and studied scientifically: animal design, river basins, turbulent flow, animal movement, athletics, technology evolution, and global design.

Evolution means design modifications, in time. How these changes are happening represents mechanisms, and

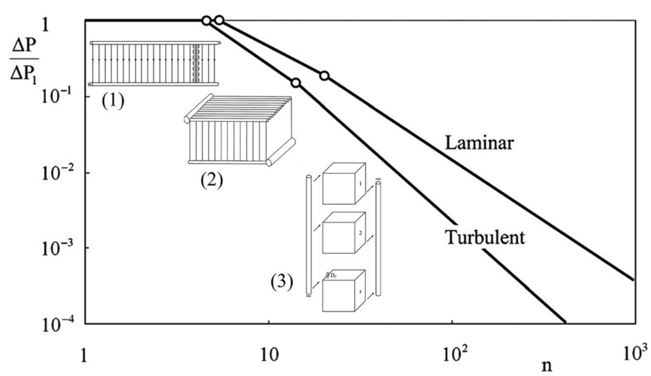


FIG. 6. The stepwise evolution of the vascular architecture as the volume size (n) increases and $d = y$ in Fig. 5.[124] Reprinted with permission from J. Appl. Phys. **107**, 114901 (2010). Copyright 2010 American Institute of Physics.

mechanisms should not be confused with principle—the Constructal law. In the evolution of biological design, the mechanism is mutations, biological selection, and survival. In geophysical design, the mechanism is soil erosion, rock dynamics, water-vegetation interaction, and wind drag. In sports evolution, the mechanism is training, recruitment, mentoring, selection, and rewards. In technology evolution, the mechanism is freedom to question, innovation, education, trade, theft, and emigration.

What flows through a design that evolves is not nearly as special in physics as how the flow system generates its configuration in time. The "how" is the physics principle—the Constructal law. The "what" are the mechanisms, and they are as diverse as the flow systems themselves. The "what" are many and the "how" is one.

"Animal design" was recognized in biology before the arrival of the Constructal law. Its chief proponents were Schmidt-Nielsen,[157] Weibel,[158] Vogel,[159] and their coworkers.[160] What was missing was the physics principle that governs animal design, and justifies the scientific approach to it. Four years after stating the Constructal law, Bejan[29] showed that the basic scaling laws of flying animals are consequences of the constructal evolutionary design toward moving animal mass more easily on the world map. The flying speed must be a certain multiple of the body mass raised to the power 1/6, or the body length scale raised to the power 1/2. The frequency of body movement (wing flapping) must be a certain multiple of the body mass raised to the power $-1/6$, or the body length scale raised to the power $-1/2$.

The work spent on flying must be proportional to the body weight times the distance traveled, just like the work spent by any other vehicle. Noteworthy is that these constructal design rules cannot be described based on the metabolic model proposed 1 year after the Constructal law by West and coworkers.[161,162]

In September 2004, Weibel and Hoppeler convened in Ascona, Switzerland, a workshop on animal design and the theory that supports animal design. Bejan[163] presented the constructal theory of animal flying. As a follow up to this workshop, Bejan and Marden[164,165] extended the Constructal theory of flying to animal running and swimming. They discovered that the body mass scaling that governs flying also governs running and swimming. For example, the frequency of leg stride and fish tailing is the same multiple of the body mass raised to the power $-1/6$ as the frequency of wing flapping.

In order to swim forward, the swimmer must lift weight, just like the runner and the flyer. Broadly speaking, all animals are weight lifters, the larger the stronger. This is why on an average the animal force is twice the animal body weight, for all flyers, runners, and swimmers. The useful energy (work, or exergy) spent by all animals is equal to the body weight times the distance traveled, times a factor that depends on the medium. That factor is of order 1 for swimmers, 1/10 for flyers, and in between for runners.

In sum, animal movement on the landscape is one design, and it does not differ in the least from the movement of all other mass movers such as the rivers, our vehicles, the oceanic currents, and the winds.

Reductionism is not the answer to predicting animal design and design in nature in general. Understanding the parts is of course necessary, but it does not lead to predicting the whole. The Constructal law runs against reductionism and empowers science to predict the design and performance of the whole. Only by knowing the architecture of how the parts flow together is one able to see the whole. One must know *the principle of construction* if one is to understand the whole, and to predict the design of the whole at larger or smaller scales.

This theoretical view of animal design is attracting interest.[163–170] One interesting facet of this field is the similarity with water waves: Their speeds obey the same proportionality with the length scale raised to the power 1/2, like all swimmers, runners, and flyers. From this observation came the realization that animal movement, like all water waves, is a "falling forward" motion. Another consequence of this line of thought is that human running, like animal running, is a wheel turning and touching the ground on just two spokes: the wheel invented by nature long before civilized humans invented the wheel, and as a manifestation of the Constructal law of evolutionary design toward easier movement on the landscape.[151]

Another interesting facet of the constructal design of animal locomotion is its real-life presence in the evolution of sports. The winning athletes in speed running and swimming are a carefully selected group of specimens, and their design evolution is driven by a single objective: speed. This objective coincides with the quality that spells success for animal species evolution, because speed empowers the predator to catch the prey, and the prey to avoid the predator. Charles and Bejan[171] showed that during the 100 years of modern athletics the speed records in the 100 m sprint and 100 m freestyle have increased along with the body sizes of the winning athletes. Size (mass, height) makes speed. The speed-mass relationships for sprint and swimming turned out to be statistically the same as for all running and swimming animals.[164,165] The same design evolution is responsible for the more recent pattern that the fastest athletes in sprint tend to be black (of West African origin) and the fastest in swimming are white (of European origin).[172] In groups of fastest sprinters with the same height, the West African is taller than the European because his or her center of mass is roughly 3% higher. This translates into a 1.5% advantage in speed on land, and a 1.5% speed disadvantage in speed in water. The Europeans have the longer torsos, and this means that they raise their bodies (shoulders) higher above the water, they generate bigger waves, and bodies and waves fall forward faster.

Running and swimming "tall" is the constructal-design route to speed in athletics. This was made clear in a theoretical paper[173] predicting the evolutionary design of swimmers toward spreading the fingers and toes (Fig. 7). Swimming with spread fingers is like wearing a glove of water boundary layers (Fig. 8)—a glove of water "stuck" to the fingers. This glove permits the swimmer to push the water downward with a greater force, and to raise his or her body higher above the water line. In the falling-forward motion that swimming is, from height comes speed.

In conclusion, by predicting the evolution of speed sports the Constructal law visualizes the evolutionary design of animal swimming, running, and flying. It predicts why the

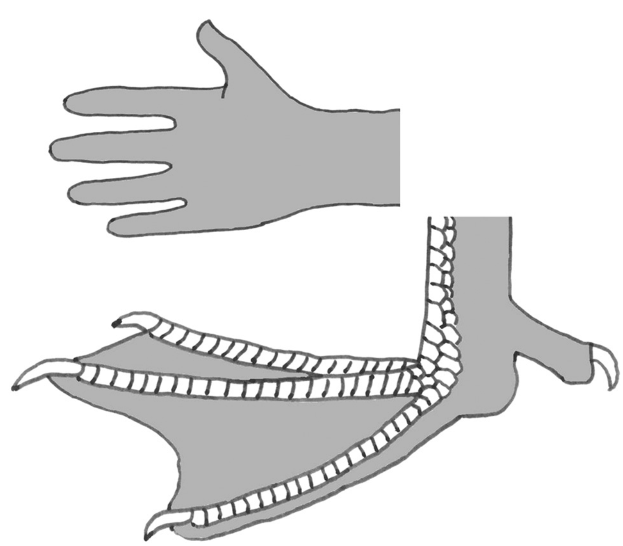

FIG. 7. Swimmers spread their fingers and toes in order to swim faster.

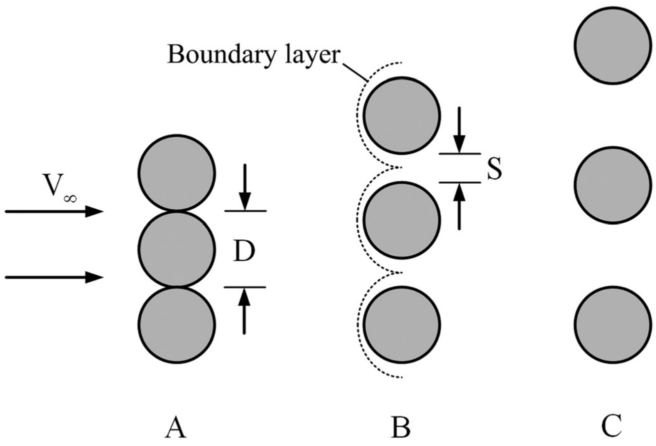


FIG. 8. When the spacing between fingers matches the thickness of the boundary layers that coat the fingers, the fingers and their "water glove" make a bigger palm that steps on water with a greater force, lifts the body higher above the water line, and gives the swimmer a greater speed.

larger animal should be faster, stronger, and with a lower frequency of body motion. The same principle dictates that the larger animal should live longer and travel a longer distance during its lifetime.[174] The life span t should scale as the body mass M raised to the power 1/4, and the lifetime travel L should scale as $M^{5/12}$. This effect of size on life span and life travel holds not only for animals but also for everything else that moves on the landscape: our vehicles, the rivers, and the atmospheric and oceanic currents, Fig. 9.

## VII. GEOPHYSICS

The flow of human constructal-design patterns at the global scale (cf. Secs. VIII–X) goes hand in glove with inanimate flow patterns of the same nature. The oneness of the animate and inanimate phenomena of design generation and evolution was stressed from the beginning in the constructal-law field.[1,2,29] Because of the theoretical progress made with the Constructal law during its first ten years, this unitary design of the animate and the inanimate was reviewed by several authors.[22,24,175–181]

The newest progress is evident in geophysics. The Constructal law was used as the physics basis for the movement of tectonic plates,[182] beach configuration (slope and sand size),[183] the construction rules (sizes, numbers) of all river basins,[184,185] particle sedimentation,[186] and the hydraulic conductivity of flow through unsaturated soil.[187,188] Broader views of the emergence and persistence (life) of flow designs at the global scale were developed by Miguel,[189] Konings *et al.*,[190] and Philips.[191] The main features of global climate were predicted based on the same principle for the steady state[192] and for climate change.[193,194]

## VIII. FEW LARGE AND MANY SMALL

The few large and many small flow together, because this is how movement is facilitated the most on an area.[152] The movement of goods has evolved into a tapestry of few large roads and many small streets (Fig. 10), and few large trucks and many small vehicles (Fig. 11). The few large and many small is also the design of all animal mass flow on the landscape. In biology and common language this is better known as the food chain, the fast catches the small, and the large eats the small (which is correct, because the larger animals are faster, on land, in water, and in the air, cf. Sec. VI).

Few large and many small are all the streams that sweep the globe. They are hierarchical, like a circulatory system with one heart with two chambers, Europe and North America.[17] Fuel consumption, economic activity, and wealth (Sec. IX) are other names for this natural design. This mental viewing has implications in the design needed by the underdeveloped to move more, to have better roads, education, information, economies, peace, and security. How is this to be done? By attaching all the areas and groups better (with better flowing channels placed in better locations) to the trunks and big branches of the flow of economy on the globe. For these attachments to flow, the grand design needs the big rivers. It needs the advanced. This is how to control the size of the gap between the developed and the underdeveloped, so that the whole design is efficient, stable, and beneficial to all its live components.

## IX. ECONOMICS AS PHYSICS

Things move and flow because they are driven. At the scale of the earth, all the flows are driven by the heat engine

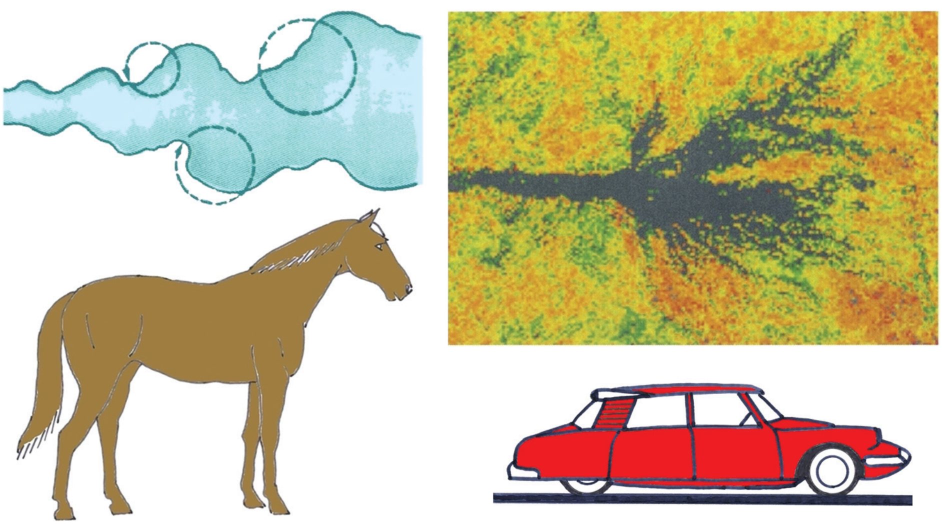

FIG. 9. The bigger live longer and travel farther: animals, vehicles, rivers, and the winds. The life span (t) and life travel (L) of animate and inanimate bodies scale as $M^{1/4}$ and $M^{5/12}$. The upper-right photo shows the Okavango river delta (NASA photo).

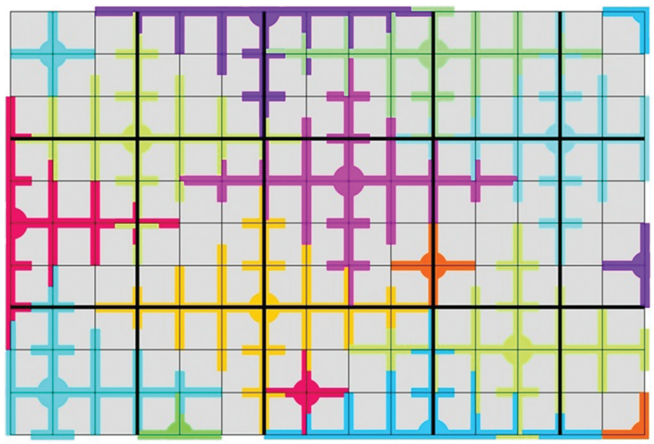

FIG. 10. The movement on the landscape appears complicated because it leaves marks (paths) that crisscross and form grids. This is particularly evident in the evolving designs of urban traffic. Less evident is the actual flow of people and goods on the area. Each flow is tree shaped, from the area to the point of interest or from another point to the same area. The grid is the solid (but not permanent) infrastructure that accommodates all the possible and superimposed tree-shaped flows. The superposition of the big branches of the trees forms the grid of avenues and highways. The superposition of the tree canopies forms the grid of streets and alleys. The few large and many small of urban design has its origin in the natural design of tree-shaped flow on the landscape. Courtesy of Dr. Erdal Cetkin, Duke University.

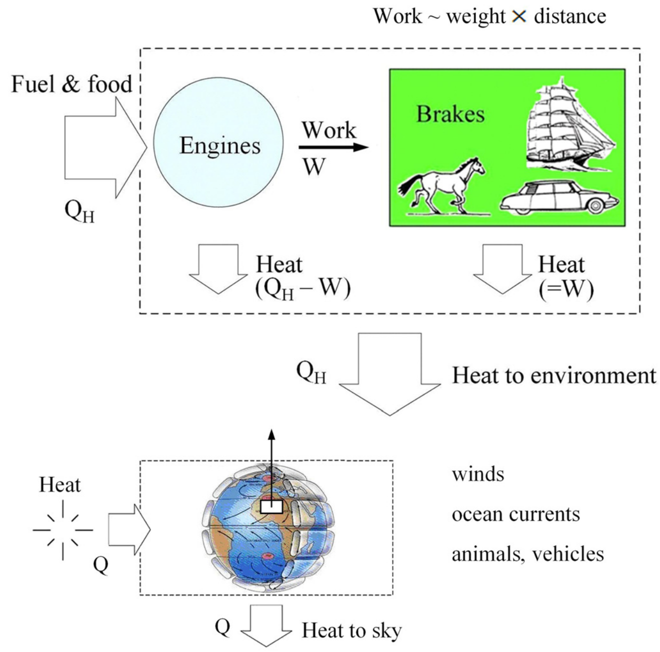


FIG. 12. Everything that moves on earth is driven. It moves because an engine dissipates its work output into a brake. Courtesy of Dr. Erdal Cetkin, Duke University.

that operates between heating from the sun and heat rejection to the cold sky (Fig. 12). Work is produced by the global heat engine, but there is no taker for this work. Instead, all the work is dissipated (destroyed) into heat, in the "brakes" shown in the figure. The net effect of the flow of heat from hot to cold is movement, namely, atmospheric and oceanic currents, and animal and human movement.

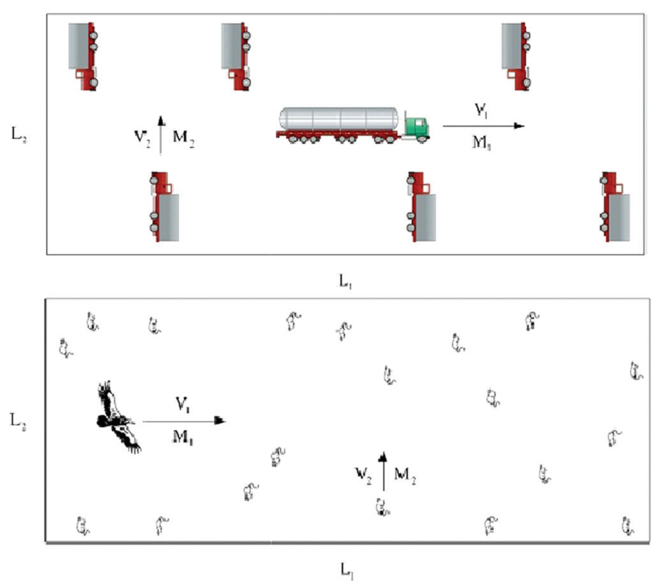


FIG. 11. Few large and many small in the movement of freight on vehicles on the landscape. The movement is enhanced when a certain balance is established between the number of small vehicles allocated to a large vehicle, and the balance ($L_1/L_2$) between the distances ($L_1$, $L_2$) traveled by the few and the many. Few large and many small in how animal mass is moving on the globe, on land, in water, and in the air. The design of animal mass flow is the precursor to our own design as human and machine species (humans and vehicles) sweeping the globe. Courtesy of Dr. Erdal Cetkin, Duke University.

All the human needs are reducible to this thermodynamic conclusion. The need to have heating, i.e., a room temperature above the ambient temperature, requires the flow of heat from the fire to the ambient. The better we configure this heat flow, the more the heat flow passes through our living space before it is dumped into the ambient. The need to have air conditioning and refrigerated spaces to store food is satisfied in the same manner. It is all about facilitating our movement on the landscape and increasing our staying power.

Completely analogous is the need to have fresh water flowing through the living space.[195] The building of infrastructure for water delivery and removal requires work, which comes from power plants that consume fuel. The need to have food (another water stream into the living space) is met through agriculture and irrigation, which require work. In the arid and populated regions of the globe, the water supply comes largely from desalination. This too requires work from fuel.

All together, the needs that define modern living are streams driven by work, or power. In time, these streams swell as the society becomes more advanced, civilized, and affluent. Better living conditions (food, water, heating, cooling) are achieved not only through the use of more fuel but also through configuring better designs (i.e., science and technology) for all the things that flow and move. We see this most clearly in the comparison of countries according to wealth (GDP, Gross Domestic Product) and fuel consumption (Fig. 13). Wealth is power,[196–198] literally, the power used to drive all the currents on the landscape, which together constitute the economic activity. The need to have water is the need to have power.

Movement and flow mean "work ~ weight × distance," as highlighted in Fig. 12. This summarizing formula holds for the work needed to drive the water flow through all the river channels, and the animals on all their paths on the

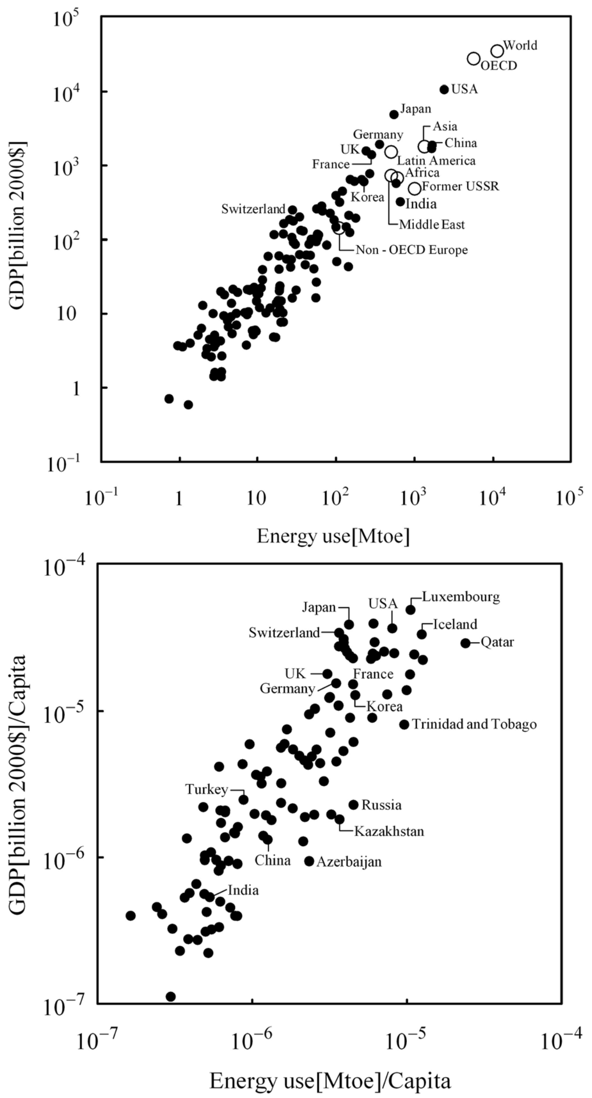


FIG. 13. Economic activity means movement, which comes from the burning of fuel for human needs. This is demonstrated by the annual GDP of countries all over the globe, which is proportional to the fuel burned in those countries (data from International Energy Agency, Key World Energy Statistics, 2006). In time, all the countries are racing up and to the right, on the bisector.

world map. It holds not only for the inanimate and animate weight that sweeps the globe horizontally but also for the weight of humanity—all the people, goods, and communications (assemblies of people and goods moving together, globally). We see the actual flow of our human and machine species in the air traffic system, which shows the design that has emerged naturally to facilitate human movement on earth. This global river basin—this vasculature—is driven by the burning of fuel. Like the movement that it drives, the burning of fuel is nonuniform and hierarchical—a few large channels and many small channels, just like the architecture of river basins, vascular tissues, and the animal kingdom.

Wealth means movement. Fuel that drives our flows is wealth (Fig. 13), because it sustains the movement of people and goods, in accord with the constructal-law tendency to morph to move more easily. The physical relation between fuel use, wealth, and sustained movement is also responsible for the relation between wealth, life expectancy, happiness, and freedom (Figs. 14–17). With the Constructal law, biology and economics become like physics—law-based, exact, and predictable.[199]

The burning of fuel and the resulting movement are not the only streams that represent wealth. There is also the creation of knowledge (science, education, information), technology, and paths of communication. *Knowledge leads to better design changes*. These morphing flow architectures happen because they are integral parts of the design of moving people and goods more effectively. They guide the process of changing and improving the design, to flow better. The flow of knowledge is an integral part of the material flow architecture on the globe, and it also means wealth—more, farther, more efficiently, all measurable in physics.[17]

Cost, or money spent, is not "energy embodied" in a product. Money is not energy. Power plants are not fueled by banks. Goods transacted, i.e., given out by some (a) and received by others (b), is a money-written record of a physical flow that proceeded from (a) to (b). Economics and business are (or, better, should be) about the accounting of the physical flows of humanity on the world map. Economics and business are about flow geography—the live flow architecture of humanity.

The global flow system is a tapestry of nodes of production embedded in areas populated by users and environment, distributing and collecting flow systems, all linked, and sweeping the earth with their movement. Constructal theory and design[9] are showing that the whole basin is flowing better (with fewer obstacles globally) when the production nodes and the channels are allocated in certain ways to the covered areas (the environment). This is how the inhabited globe becomes a live system—a living tissue—and why its best future can be designed based on principle. With the constructal law, this design can be pursued predictively.

Once again, few large and many small is the secret of the global design, because we showed recently that larger flow systems must be more efficient than smaller systems.[110,152] This is in accord with the recorded performance of steam turbine power plants, gas turbine power plants,

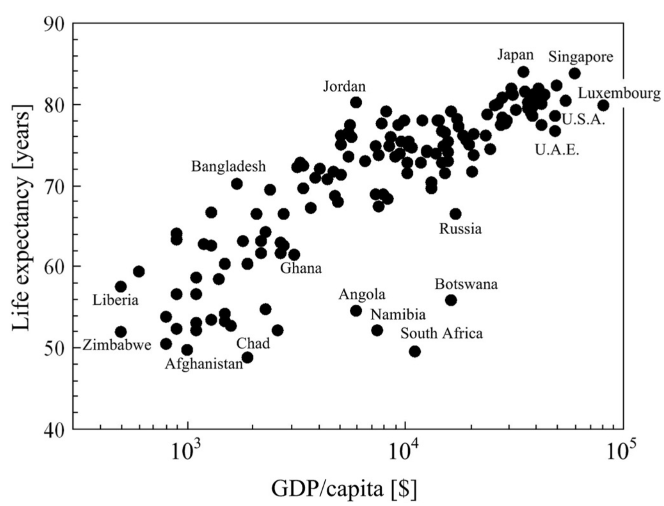


FIG. 14. More economic activity also means longer life span (data from CIA World Factbook).

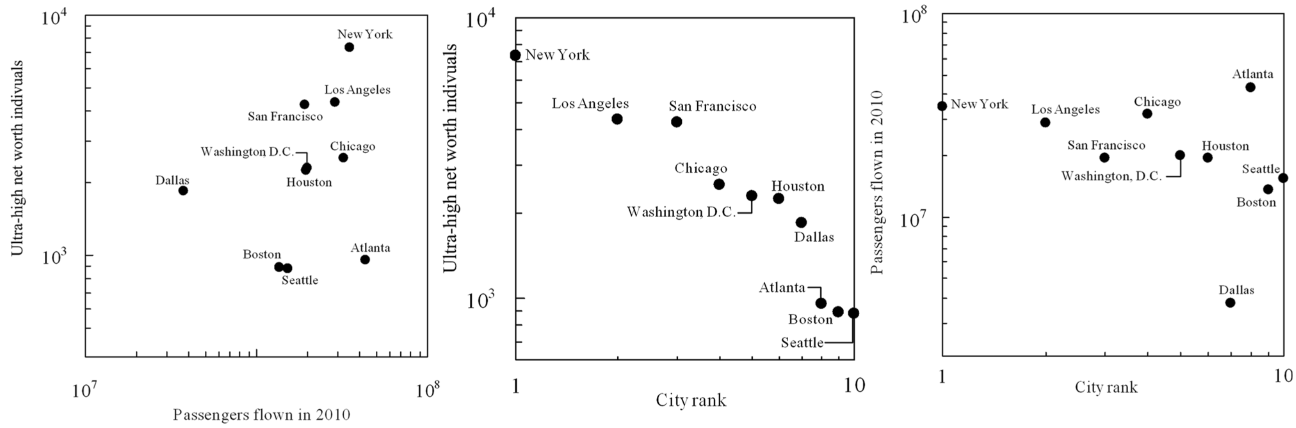


FIG. 15. Wealth means movement: the rankings of cities according to air traffic and wealthiest inhabitants (data from Wealth-X and Bureau of Transportation Statistics, T-100 market).

individual turbines, and refrigeration and liquefaction installations.[200] The size effect is predictable from the argument that larger body sizes accommodate ducts with larger cross-sections and larger surfaces for heat and mass transfer, which represent lower resistances for the flow of fluids and heat and mass currents. This holds equally for power plants and animal design, and is summarized in an efficiency formula of the form $\eta_{II} = C_1 M^{\alpha}$, where $\eta_{II}$ is the second law efficiency, $C_1$ is a constant, M is the body mass, and $\alpha$ is in the range 2/3–3/4. This also predicts the efficiency of animal design as a transportation system for animal mass on the landscape.

This effect of "economies of scale" can be predicted by considering even simpler flow systems. We showed that when heating water in a central facility the loss of heat from the water mass m is proportional to the surface of the water tank (i.e., $m^{2/3}$), and consequently the heat loss per unit of heated water mass decreases as $m^{-1/3}$ as the body size m increases.[9] Another example is the power output of solar chimney power plants, which increases in proportion with $A^{3/2}$, where A is the land area occupied by the roof of the power plant.[115,201]

At first glance, this size effect suggests that bigger is always better. We showed that this is incorrect because the efficient system must serve a population of users distributed on an area. When the system is large, the area is large and the users are many. The distribution lines that connect the central system with the users are plagued by losses that

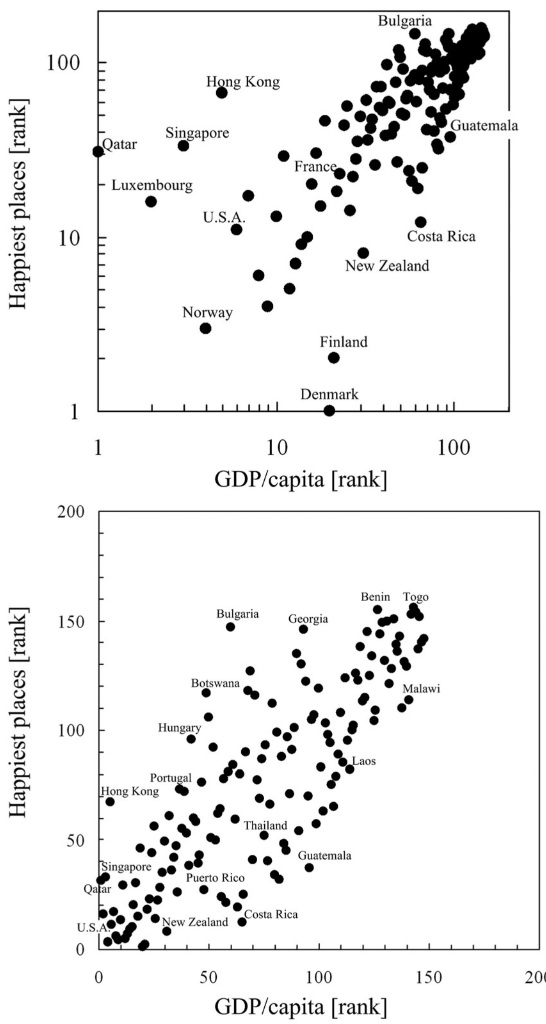


FIG. 16. Movement (wealth) is broadly understood as happiness (data from CIA World Factbook and World Happiness Report, Columbia University, 2012).

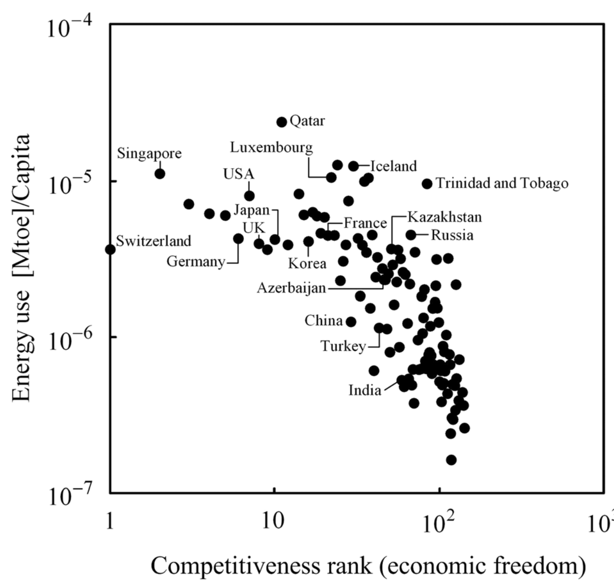


FIG. 17. The free societies have wealth and staying power. In time, all the countries are moving up (cf. Fig. 13), and this means that they are all evolving toward more freedom.

increase in proportion with the length scale of the area. From this second point of view, smaller is better. There is a fundamental tradeoff between the two effects, and its chief result is that a balance always emerges between the size of the central system and the number of users that the system serves. We demonstrated this for the generating and distribution of power on a land area,[201,202] refrigeration and air conditioning,[203] heating,[204] and education.[205–207]

The landscape emerges as a tapestry of nodes of production and lines of distribution. The nodes are few and large, and the branches that reach the users are many and small. We also discovered that this tapestry must be woven according to a vascular design that depends on the size of the whole system. For example, while distributing heated water from a central heater to a square area with N uniformly distributed users, the flow architecture can be radial (r), dichotomous (2), or a construct (4) based on a quadrupling rule, Fig. 18. The lower part of the figure shows that the total heat loss per user (i.e., the loss at the center and along the distribution lines) decreases as the size of the landscape (N) increases. In the pursuit of efficiency (less fuel required per user), the flow architecture must change stepwise from (r) to (2), and finally to (4) as the overall size increases. The *stepwise* evolutionary design of vasculature covers all scales, including the water and energy design of the inhabited globe, cf. Fig. 6.

## X. GOVERNMENT

To improve government is and has always been of paramount importance. Just look at the situation in which our country and the whole world is today. The evolutionary-design path to better government is spelled out in a law of

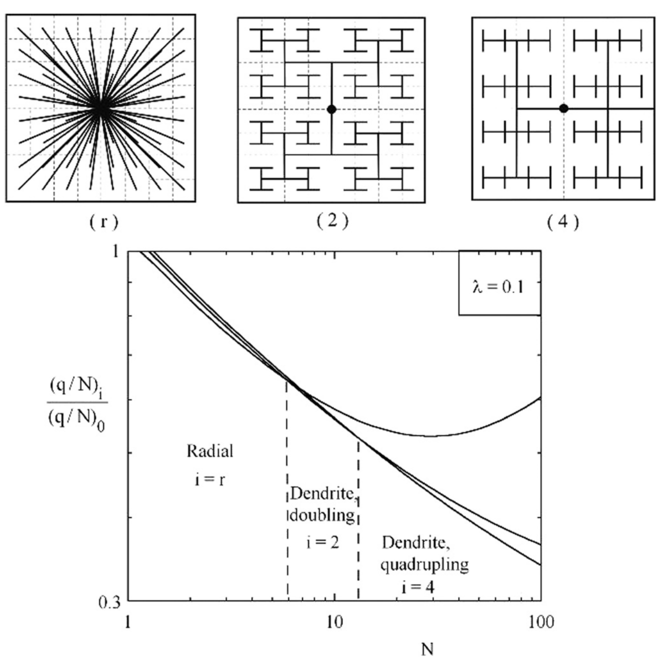


FIG. 18. The effect of size on the design of distributed heating on the landscape. The total heat loss per user decreases as the size of the inhabited area increases. The heat loss per user is lower when the architecture evolves stepwise from radial to dendritic as N increases.[204] Reprinted with permission from J. Appl. Phys. **108**, 124904 (2010). Copyright 2010 American Institute of Physics.

physics: the Constructal law of design and evolution in nature. This law is about a universal phenomenon: Everything that flows and moves does so with evolutionary design, which means changing flow configurations, patterns, and rhythms that morph freely over time to provide greater access to their streams, to flow more and more easily.

This natural tendency is the time arrow of design evolution. We see it everywhere, in animate and inanimate flow systems, from the birth of river basins and the growth of snowflakes, to lungs, vascular tissues, animal migration and urban and air traffic.

The rule of law and the government are descriptions of our own movement "with design" on earth. The traffic signs in the city are just one example. They all "happen," and their evolution toward easier flowing over time happens. This is the time arrow—the history—of our civilization.

Civilization also means engineering science, which is the ability to design a better future, and to walk into it with confidence. Therefore, instead of waiting a long time for better government to happen, we can rely on the Constructal law to fast-forward the evolution toward better government.

How? By opening up the channels through which we and our belongings and our associates move on the entire earth. This means to shorten, to straighten and to smooth all the channels, to remove the obstacles, the bottlenecks and the checkpoints, and to minimize the tediousness that frustrates every single one of us every day.

See all of us as who we are: We are a river basin of movers who go with the flow, and yearn for easier and freer movement. Easier movement means many things: greater efficiency, getting smarter, wealthier, and a better economic sense in each of us.

In order for a flow design to change, the design must have freedom to change, to morph, to evolve. River deltas carved every day in the silt have freedom, and every day they display the best flowing design, which is a tree better than yesterday's tree.

Freedom endows all the flowing designs with two things: efficiency and staying power (Fig. 17). This is why social systems that are free to change have two characteristics—wealth and longevity. Rigid systems have the complete opposites—poverty and catastrophic change. Without freedom, changes in flow configuration (design changes, evolution) cannot happen. The evolution of government toward a more open government is the evolution toward freedom, wealth, and longevity.

Technology, science, information, education—in one word, culture—is how all of us unwittingly open up our channels and liberate our flows. As Professor Vadasz observed,[208] "any society has as much freedom as the available technology can provide and support." This is why design science is so important and valuable, and why the Constructal law teaches how to fast-forward the design of open government. The Constructal law has been applied to the design of currency market dynamics,[209] digital governance,[210] and warfare.[211]

We all share the need to see how government works, how it changes to work better, and how it could be designed to change faster toward getting better. To describe all this, we need an unambiguous understanding of the terms that we use. We need a narrative that makes sense to the largest audience. The Constructal Law provides the physics basis for defining the terms. It places in palpable terms many intangibles such as government, freedom, business, wealth, data, knowledge, information, and intelligence.

Government is a complex of rules and channels that guide and facilitate the movement of humanity (people and goods) on the world map. Many individuals are employed in government in order to construct, maintain, and change the rules and the channels. They are "employed" because they are physically engaged in (i.e., they are part of) the entire flow system of humanity. This engagement is what drives the employees' own movement on the landscape, and why they too have a stake in improving the flow design, and why they go with the flow. The flow system of humanity has the built-in capability of generation, maintenance, and evolution of flow architectures that make the whole flow better.

To see this in physical terms, think of evolution of urban design and city traffic. Look through old telephone books and compare the maps of your city over the past few decades. These designs "happened" because of the urges of all the inhabitants. They are not God given. They are not the wish of one person. They are forever imperfect, inviting changes in the strangled channels, and no changes in the channels that flow with ease (as in the saying "if it ain't broke, don't fix it"). Like the ant mound, the city design emerges and evolves naturally in a particular direction over time because it empowers every inhabitant. The city design is the physical version of intangibles, such as "the wisdom of the crowd" and the "wisdom of the ants."

Flow generates better flow. A society that flows more is wealthier, cf. Fig. 13. It has a greater tendency to reconfigure itself to flow even more, and to become even wealthier over time. There is no end to this evolving design. There is just the time direction of the evolutionary changes, and the rate at which changes are occurring.

Good is a government that facilitates the movement, the reach and the staying power of the whole society, which include mobility, participation, access, health, and life expectancy. A government becomes better when it opens the channels, shortens and straightens the paths, removes road blocks, and reduces waiting times.

Government is not the only complex of rules and channels that guide the flow of humanity. It is only the biggest, at the largest scales on the world map (country, alliance, world). Other complexes of morphing channels that facilitate our movement are business (companies), education (schools, universities), and science. The use of science in practice is technology. Science and technology are one: All science is useful.

Government, business, and technology happen. They appear out of nothing, and evolve to facilitate the flow of the whole, which is we the living. They are flow architectures that flow hand in glove so that the whole society flows better. They are like the circulatory, respiratory, and nervous systems that keep the animal body-design flowing internally and moving on the landscape.

There is no conflict between government, business, and technology. On the contrary, these designs evolve as one in order to facilitate the movement, the reach, and the longevity of each of us. Their evolution is the large-scale manifestation of every individual's urge to be free, to make choices, to vote, and to make changes to live better. The perceived conflict between government and business is due to the natural give and take between two constantly adjusting flow designs that bathe the same landscape in the same evolutionary direction, and with the same purpose. A more open government is good for better flowing business streams, and vice versa. More efficient business flow structures engage and sustain the constantly adjusting flow structures of government and the rule of law.

Centralized vs decentralized (distributed) is one design, not two. This unitary design is a vascular tissue with hierarchical flows, few large channels flowing hand-in-glove with many small channels.

Decentralized (distributed) does not mean uniform, or one size fits all. Distributed means allocated, for example, a channel (a stream) of this size allocated to an area (a population) of this size. The allocation of channels to areas over many scales is the hierarchical vasculature that bathes the area most efficiently, and empowers all the inhabitants the most.

Global vs local is one design, not two. The sizes and numbers of channels, and their placement on areas with certain sizes and numbers are the hierarchical flow design governed by the Constructal law. Theory alone enables us to see this design from the micro to the macro. Constructal law empowers us with scaling—the ability to scale up the designs that we understand and improve at smaller scales. To scale up the design, one must possess the principle on which the design is based. The large aircraft is not a magnified version of the small aircraft. The large animal is neither a magnification nor a repeated compounding (assembly) of the small animal.

Data are not knowledge. Data must not be confused with intelligence either, and "open data" must not be confused with "open government." Data is the plural of *datum* (given), i.e., something that is "in hand," known or held, i.e., a fact on which anybody can rely. The data are the facts that we accumulate based on observations, measurements, and surveillance. Making data available without the principles to decode, understand, and analyze them is pointless. This approach could reach the opposite of the effect that is sought. One could even imagine the perverted attitude of a government that would drown it citizens in an ocean of data, to the point that the most important items are hidden under superficial data. Such a government could not be accused of lack of openness, and it would be hard to question.

Today, data are flooding our mental field of vision with streams so large that they are impossible to store. For this, technology evolves toward computer memories that have greater densities and greater volumes at the same time. This trend is not new. The technology of gathering data has always been evolving toward greater streams of data, from the telescope and the microscope to spy satellites and surveillance cameras on streets and in buildings. Science has been generating open data throughout the history of civilization. Science has also been facilitating the "opening" of data, through better alphabet, better numerals, better books, tables, plots, matrices, journals, libraries, and all the physical structure that supports information storage today.

Knowledge (*scientia*, in Latin) is science, and science is both, the observing and the mental condensing and streamlining the flow of observations. The condensed are the principles, and, among them, the most unifying are the first principles, the laws.

Intelligence is the ability to effect design change. It is to use knowledge to make changes in how we move on the landscape, and how we rearrange the landscape. Intelligence is to "see" a better design before the better design is spoken, tested, and built. Intelligence is the fast forwarding of design generation and evolution.

## XI. THE S CURVE PHENOMENA: SPREADING AND COLLECTING

The flows that bathe and connect the live landscape are united not only by the tapestry of tree-shaped flows (Figs. 10 and 11) but also by the unsteady (nonmonotonic) manner in which these flow architectures spread. When the covered territory is plotted versus time, its history is an S-shaped curve.[212] In the beginning the covered territory grows slowly, but the rate of territory coverage increases in time. The growth rate is maximum at the point where the S-curve is the steepest. Later, the growth rate decreases monotonically and the covered territory tends to a plateau, which is the upper end of the S (see the examples of Fig. 19).

To see how to predict the S-curve phenomenon, consider the operation of a heat pump coupled to the ground for the extraction or rejection of heat.[212–215] First, the heat must be spread by fluid flow, through tree-shaped pipes, throughout the territory. During this initial "invasion" phase, the volume of the heated soil is small (Fig. 20, upper left), but it increases at a growing rate. Second, after the hot fluid has invaded all the channels on the territory, the heat is transmitted from the channels perpendicularly to the neighboring soil. This is the "consolidation" phase, where the theme "solid" in the word

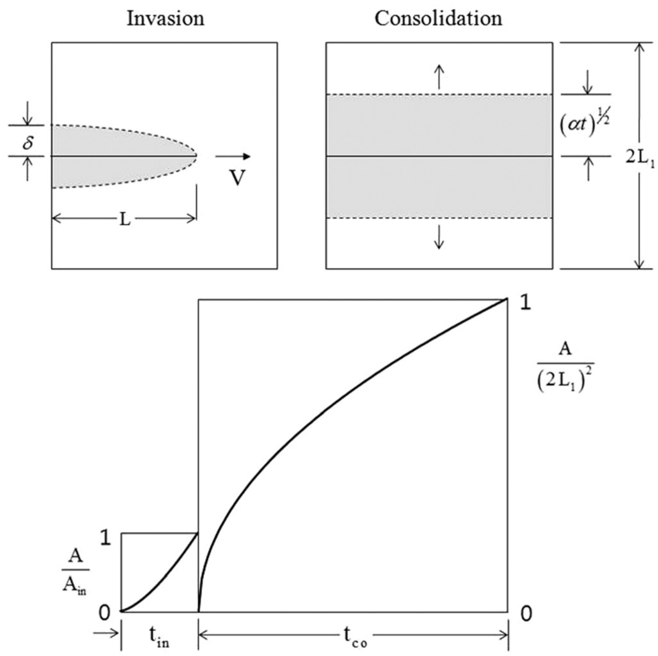


FIG. 20. Line-shaped invasion, followed by consolidation by transversal diffusion. The predicted history of the area covered by diffusion reveals the S-shape curve.[212] Reprinted with permission from J. Appl. Phys. **110**, 024901 (2011). Copyright 2011 American Institute of Physics.

consolidation suggests the reality of the heat current filling the soil interstices held between neighboring channels.

The history of the volume of heated soil versus time is an S-shaped curve (Fig. 20) that is entirely deterministic, i.e., predictable. Everything about this S curve is known because both phases, the invasion and the consolidation, are known. During invasion the covered area increases as $t^{3/2}$, not as an exponential. This initial phase is not "explosive." During consolidation, the covered area increases as $t^{1/2}$. When the invading channels are tree-shaped (Fig. 21), as opposed to single channels (Fig. 20), then in accord with the Constructal law the entire flow from the point to the volume occurs faster, more easily, along a steeper S curve.[215]

In summary, and in accord with the Constructal law, the S curves of nature are manifestations (history records) of tree-shaped invasion (not line invasion) on areas and volumes that are eventually filled during consolidation by transversal diffusion. This discovery of the S curve is important for two reasons. First, the S curve was predicted from the constructal law before there was any reason to look outside to see many diverse S curves and try to predict them in order to unite them. This first part of the story is about the meaning of pure theory. It is images in the mind, in that imaginary movie theater.

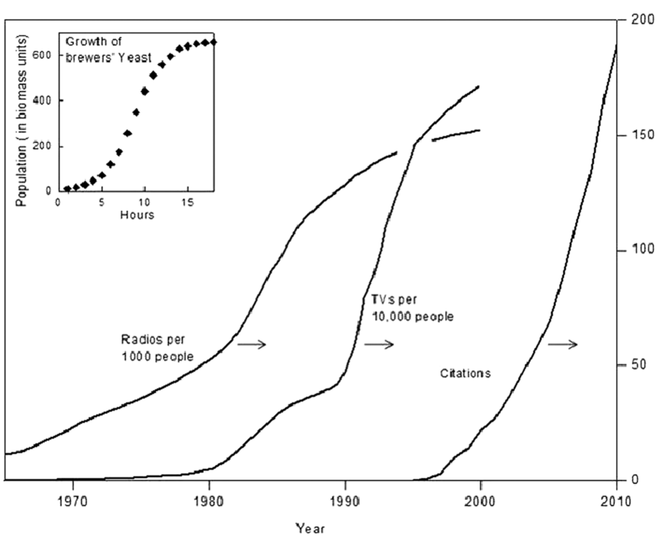


FIG. 19. Examples of S-curve phenomena: the growth of brewer's yeast, the spreading of radios and TVs, and the growth of the readership of scientific publications.[212] Reprinted with permission from J. Appl. Phys. **110**, 024901 (2011). Copyright 2011 American Institute of Physics.

The second part is about the practical value of this power to predict. When anything spreads on a territory, the curve of territory size vs. time is S-shaped: Slow initial growth is followed by much faster growth, and finally by slow growth again. The corresponding curve of the rate of spreading vs. time is bell shaped. This phenomenon is so common that it has generated entire fields of research that seem unrelated: the spreading of biological populations, cancer tumors, chemical reactions, contaminants, languages, news, information,

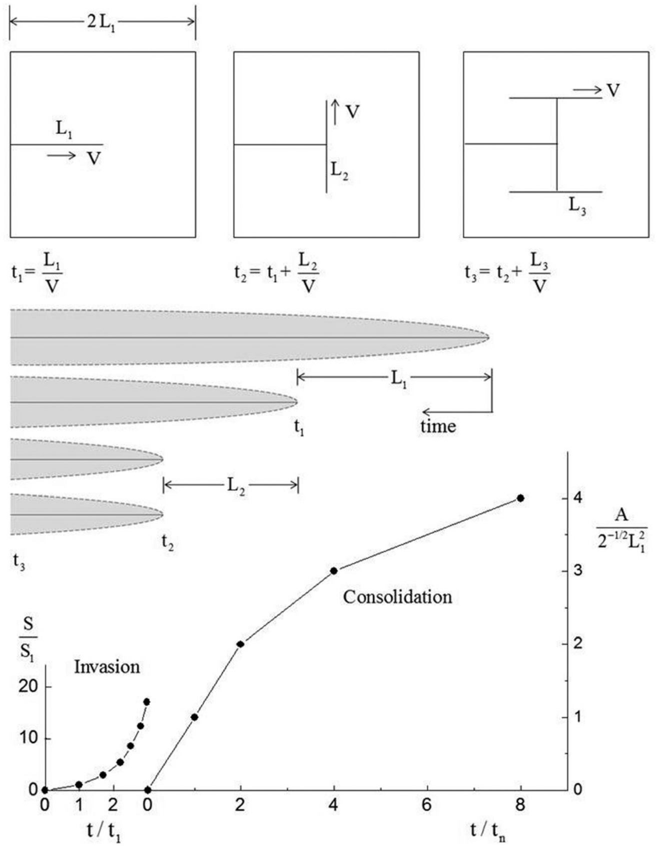


FIG. 21. Tree-shaped invasion, showing the narrow regions covered by diffusion in the immediate vicinity of the invasion lines.[212] Reprinted with permission from J. Appl. Phys. **110**, 024901 (2011). Copyright 2011 American Institute of Physics.

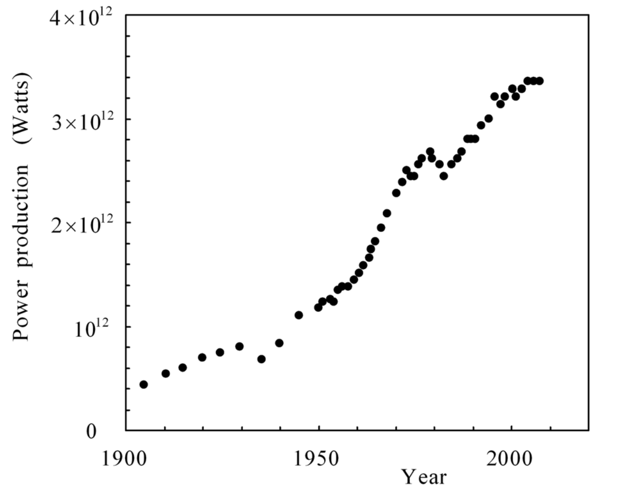


FIG. 22. The S-shaped history of power generation in the U.S. during the 20th century (data from EIA/AER, Annual Energy Review 2003, Energy Information Administration, U.S. Department of Energy, Report No. DOE/EIA-0384(2003), 2004).

innovations, technologies, infrastructure, and economic activity (e.g., Figs. 19 and 22).

The spreading of ideas exhibits the same S-curve phenomenon. This is illustrated by the history of citations registered by every scientific publication, and by the total number of citations of a single author during his or her career.[216] We showed that as a consequence of this history the h index of every author also traces an S-shaped curve during the author's life of creative work.

Collecting flows such as oil extraction and mining cover their available areas and volumes while exhibiting S-curve histories. In the field of oil extraction, the steepest portion of the S curve is known as the Hubbert peak. In accord with the progress from Figs. 20 to 21, oil extraction and mining are evolving toward dendritic wells and mine galleries, because trees offer the steeper S curves than straight shafts.[215]

The natural tendency that drives all the S-shaped histories of invasion and retreat on the landscape also governs the movement of people and animals.[217–220] For example, Lui *et al*.[218] showed that the evacuation of pedestrians from public places can be made to occur faster by properly sizing and shaping the floor areas and bifurcations of walkways.

## XII. CONCLUSIONS

The fast growth of the Constructal-law field, which is documented in this review article, is an illustration of the much broader phenomenon of how and why science evolves and improves. Science is an evolutionary design in which what we know—what is true, what works—becomes simpler, more accessible, and easier to teach.[207]

The Constructal law is a new law of physics that broadens significantly the reach of thermodynamics.[221] The merger of mechanics with caloric theory into thermodynamics in 1851 was not the end of this morphing by simplification and replacement. The caloric line continued to this day as thermometry, calorimetry, and heat transfer (Fig. 23). Although mechanics and caloric theory were incorporated in

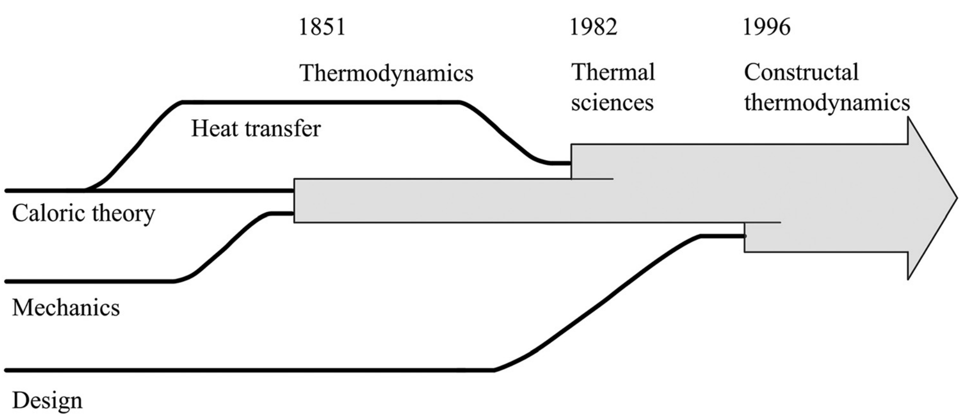


FIG. 23. The evolution and spreading of thermodynamics during the past two centuries (after Ref. 19, Diagram 1, p. viii).

thermodynamics, heat transfer developed into a self-standing discipline, with major impact on applied mathematics, fluid mechanics, and aerodynamics. Still, its proper place is in thermodynamics along with all the other caloric teachings.

The merger of heat transfer with thermodynamics was predicted in 1982 in the preface to Ref. 19 (Fig. 23), and the prediction came true in the two decades that followed. Heat transfer journals became journals of "thermal sciences" (which means heat transfer + thermodynamics), and in many universities the heat transfer and thermodynamics courses were combined into a single course on thermal sciences.

Thermal sciences expanded in new directions, most vigorously now because of the Constructal law, which unifies science (physics, biology, engineering, social sciences). Constructal thermodynamics[221] places the concepts of life, design, and evolution in physics. It constitutes a wide open door to new advances especially in areas where design evolution is key to performance, for example, in logistics,[222] biological evolution,[223] art,[224] and business and economics.[225]

Constructal thermodynamics claims a role for design, configuration, and geometry in understanding the language of nature.[226] The Constructal law runs against reductionism, and empowers the mind to see the whole, its design, performance, and future. In modern times, physics grew on a course tailored to infinitesimal effects. The Constructal law is a jolt the other way, a means to rationalize macroscopic design, objective, and behavior.[29]

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