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Fractal Trigeometry and the Juliusbulb Iteration

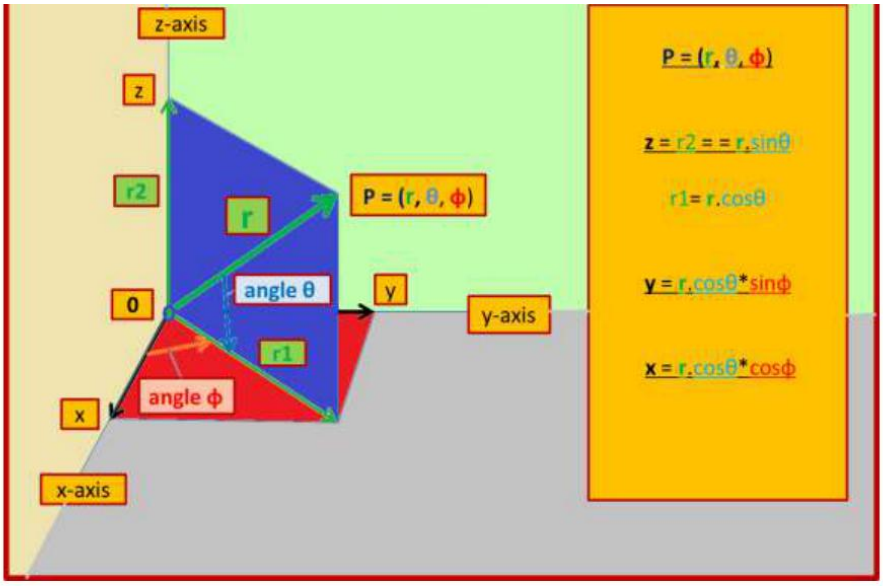
A Coordinate-Based Geometric Dynamical System in $\mathbb{R}^3$

Abstract

Fractal Trigeometry (FTG) is formulated as a coordinate-based geometric extension of polar complex dynamics to three-dimensional Euclidean space. Rather than defining a hypercomplex algebra on $\mathbb{R}^3$, FTG introduces exponentiation through radial scaling and angular multiplication in spherical coordinates. The resulting mapping induces a well-defined discrete dynamical system whose bounded orbits define three-dimensional analogues of Julia and Mandelbrot sets, referred to as the Juliabulb and Juliusbulb.

1. Coordinate Framework

Let $z = (x, y, z) \in \mathbb{R}^3$. Define $r_1 = \sqrt{(x^2 + y^2)}$, $r = \sqrt{(x^2 + y^2 + z^2)}$, $\varphi = \text{atan2}(y, x)$, $\theta = \text{atan2}(z, r_1)$. The inverse relations are $x = r \cos\theta \cos\varphi$, $y = r \cos\theta \sin\varphi$, $z = r \sin\theta$.



3. Juliusbulb-(p, n) notation

With p is the power of z and n is the number of iterations.

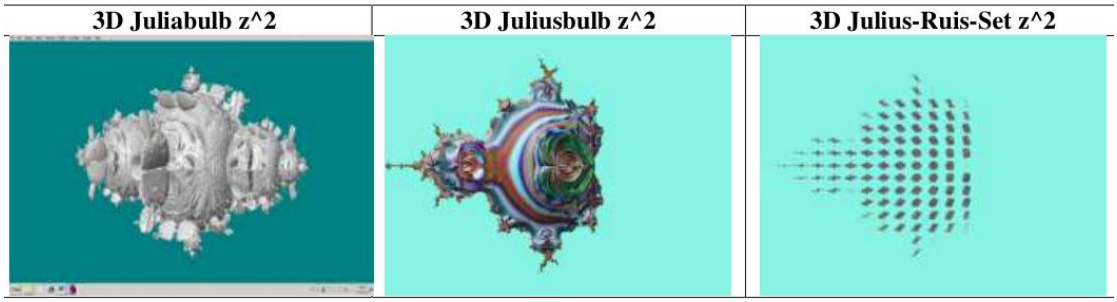
2. Geometric Power Mapping

In $\mathbb{C}$, exponentiation is given by $(re^{i\varphi})^p = r^p e^{ip\varphi}$. Since no associative multiplication on $\mathbb{R}^3$ extends this structure, FTG defines exponentiation geometrically.

Definition (FTG Power Map). For $p \in \mathbb{R}$ define $F_p : \mathbb{R}^3 \setminus \{0\} \to \mathbb{R}^3$ by $F_p(x,y,z) = r^p (\cos(p\varphi)\cos(p\theta), \sin(p\varphi)\cos(p\theta), \sin(p\theta))$.

3. Dynamical System

For $c \in \mathbb{R}^3$ define the iteration $z_{k+1} = F_p(z_k) + c$, $k \geq 0$. This defines a discrete nonlinear dynamical system on $\mathbb{R}^3$.



4. Summary Formula (FTG Canon)

Given: $z_0 = (x, y, z)$

4. Boundedness and Escape

Let $R > 0$. An orbit is bounded if $||z_k|| \leq R$ for all k. The escape time is $n(z_0) = \min\{k : ||z_k|| > R\}$.

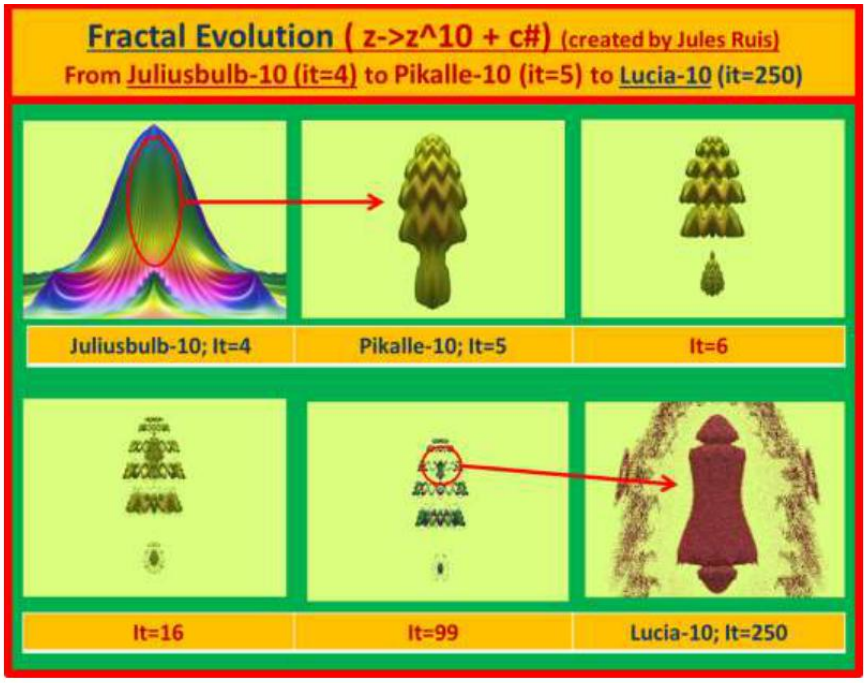
Historical Origin of the Juliusbulb Formula

5. Fractal Sets

Juliabulb: $J_p(c) = \{z_0 \in \mathbb{R}^3 \mid \text{orbit bounded}\}$.
Juliusbulb: $M_p = \{c \in \mathbb{R}^3 \mid \text{orbit bounded for } z_0 = 0\}$.

6. Numerical Parameterisation

Notation Juliusbulb-(p,n) denotes truncation after n iterations, where p controls nonlinearity and symmetry and n controls numerical resolution only.



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7. Relation to Existing Work

FTG belongs to the class of geometric generalisations of complex dynamical systems. Unlike algebraic constructions, FTG defines exponentiation explicitly as a coordinate-based geometric mapping in $\mathbb{R}^3$. It is conceptually related to Mandelbulb-type constructions, while remaining mathematically independent of any hypercomplex algebra.