Fractal Geometry in Medical Science: Beginning of a New Era?

In present days, many scientists strongly have opinion that fractal geometry is a revolutionary area of mathematics which has deep impact on every branch of science including medical fields. John Archibald Wheeler rightly remarked that "No one will be considered scientifically literate tomorrow who is not familiar with fractals" [1].

But what is fractal? Why is it considered so much important? Perhaps all of us are very familiar with Euclidean geometry which has been established by Euclid of Alexandria in 300 BC. We are very much dependent of Euclidean geometry. For two thousand years we tried to describe nature with the help of Euclidean geometry. But question often arises, how far are we successful? In fact nature does not follow Euclidean geometry.

Biological systems are predominantly irregular, complex and non-linear. Irregularities of biological system cannot be quantified by means of classical Euclidean geometry. The clouds are not sphere, mountains are not cones, and lightning does not travel in straight lines. This is true for biological systems also. Can we quantify the tumor vasculature or surface of irregular complex shape of a gland? Can we describe the alveolar surface area of lung or glomerular perfusion area of kidney by Euclidean geometry?

To overcome these limitations of Euclidean geometry, Mandelbrot first time formalized the concept of fractal dimension [2]. He introduced the term "fractal geometry" and attempted to explain the behavior of chaos in nature. The fractal geometry is one of the important tools to explain true geometry of nature. In fact, this new area of mathematics enhances the power of Euclidean geometry. Euclidean geometry deals with objects in integer dimensions but fractal geometry deals with non-integer dimension. This is known as fractal dimension (FD). Fractal geometry is basically self-similar and the part of the image is exactly similar as whole of the image. This self-similarity of fractal is very important.

Our human body is the best example of fractal geometry. Our lung, brain and vascular systems are all fractal. The bronchial tree and its branching is fractal. It has particular FD. Similarly the anatomical organization of the most mysterious organ of our body, the brain, is also fractal.

Now the question comes, how to calculate fractal dimension? The simplest way to calculate FD is by taking the advantage of self-similarity. Let's take an example. Suppose we have a one dimensional line segment. If we look at it with the magnification of two, we will see two identical line segments, and $2_1 = 2$, where 1 indicates the dimension. In a two dimensional square, with the magnification of 2, we get 4 identical shapes in both of them and $2_2 = 4$ and hence 2 indicates the fractal dimension. Finally, take a three-dimensional cube and doubling its length, breadth and height, we get eight identical cubes, that is $2_3 = 8$ and 3 indicates the fractal dimension. So we can see clearly that the magnification and raises it to the power of dimension, the number of shapes is obtained. This can be expressed as

$E^{\wedge}{}_{D} = N$

Here, E stands for the magnification, D for dimension and N for the number of identical shapes. Applying logarithms, $D = \log N / \log E$

There are various ways of measurement of FD such as
1) Modified pixel dilatation
2) Perimeter-area method
3) Ruler counting method and
4) Box counting method. Box countingmethod is most commonly used for detection of FD. It is a simple and reproducible way of measuring fractal dimension [3].

Fractal geometry may be applied to the various fields of medicine such as cardiovascular system, neurobiology, pathology and molecular biology [3, 4]. Heart rate of healthy individuals in different time scale shows essentially similar pattern. This pattern is self similar and fractal. In various diseases of heart, the heart rate variability loses its complex fractal pattern [4]. So it may be possible to predict impendingarrhythmia. Fractal concept has been applied to measure the infiltrative margin of the malignant tumor [5], to assess the tumor angiogenesis [6] and also to measure irregular distribution of collagen in tissue [7].

Einstein et al mathematically described the chromatin distribution of malignant cells with the help of fractal geometry [8]. We also have noted that the irregular margin of malignant cells can best be demonstrated by fractal geometry and fractal dimension is a useful tool to separate malignant from benign cells [9, 10]. We also quantitatively measured the irregular glandular margin of simple hyperplasia, atypical hyperplasia and adenocarinoma of endometrium with the help of fractal geometry. We noted that fractal dimension is helpful to differentiate these lesions [11].

By doing multiple iterations of simple equations, a fractal image can be generated. This technique has been employed to generate various biological models [12, 13]. In future, this may help us to understand the biological processes and also to fight against cancer [14]. Fractal geometry is important in viral and bacterial infections. In fact the receptors responsible for invasion of these organisms are all fractal and their binding functions follow the deterministic rules of fractal geometry[1]. It has been noted that DNA sequences follow the fractal properties, and intron and exon sequences differ in their fractal properties. This information may be helpful to reconstruct the evolutionary history of organisms. New and efficient codon indices can be developed by simultaneously characterizing the fractal and periodic features of a DNA sequence [15]. By simple one or two instructions, one can make complicated fractal images. This suggests that DNA codes send only a few commands for developing complicated anatomy of the body such as bronchial trees or vasculature. This is important in morphogenesis.

Fractal geometry is a rapidly growing area of mathematics with immense potential. Within few years it will help us to explore the mystery of morphogenesis, tumorogenesis, angiogenesis, classifying disease entities and many unfold mystery of human life.

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