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Blood flow simulation through fractal models of circulatory system

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Abstract

The blood flow in human arteries has been analytically calculated according to Poiseuille's equation. Geometry of the fractal arterial trees has been described in previous article [Gabryś E, Rybaczuk M, Kędzia A. Fractal model of circulatory system. Symmetrical and asymmetrical approach comparison. Chaos, Solitons & Fractals, in press]. Blood vessel trees are consisted of straight, rigid cylindrical tubes. In each bifurcation two new children segments appears according to Murray law.

Blood flow in circulatory system is driven by the pressure differences at the two ends of the blood vessel. A mathematical analysis shows the continuous dependence of the solution on vessel tree parameters and boundary condition.

1. Introduction

The ability to predict the pressure and flow at any site in arteries can lead to a better understanding of the arterial function. Theoretical models play an important role in understanding the hemodynamic forces.

Although in vivo studies might give exact solutions, the results are very difficult to obtain. So far, attention has been given mainly to the fractal geometry and fractal dimension of blood vessel trees without flow analysis through fractal models [10,17–19].

Flow analysis in circulatory system is usually done for simplified model consisting of one or two bifurcation levels and relatively high vessel diameters common to arterial level. Some work concern on numerical analysis of blood flow in circulatory system is reduced to 1D or 0D model [2,7].

The description of blood flow in circulatory system with some local phenomenon (carotid bifurcation, stented artery etc.) is better described by 3D numerical simulation, based on the Navier–Stokes equations. However, from the computational point of view, numerical simulations of the 3D tree completely based on these equations are unaffordable at

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present. For given 3D fractal blood vessel trees analytical calculation of blood flow based on reduced Navier–Stokes equation has been done.

Fractal model of blood vessel system is a certain geometrical simplification but it suffices for acceptable blood flow analysis. This analysis permits understanding influence of hemodynamic forces and its role in the development of vascular diseases.

2. Modelling of the blood flow through fractal vascular tree

The liquid motion equations for circulatory system are very complicated. In order to obtain analytical solution following simplifications have been assumed.

Blood is non-Newtonian fluid consisting of blood cells and blood plasma. Proportional relation between cells and plasma is determined by hematocrit value [14]. The hematocrit value is the most important parameter defining blood viscosity. The hematocrit of normal human blood is about 45% and it relates to blood viscosity about 4×10^{-3} [Pa s]. Assumption of constant blood viscosity and homogeneity in whole vessels tree is necessary to estimate blood flow through blood vessel trees. According to researches [1] during a normal flow in straight arteries blood behaves as a near Newtonian fluid.

In real blood vessels system, vessel walls are elastic and can change its diameters. In this way resistance of blood vessel system is regulated. This process is known as autoregulation and corrects nutrition of all cells in human body. Assumption of vessel wall as a rigid pipe with constant diameter for given vessel segment is necessary to application of hydrodynamical equations and analytical calculation of modelled trees.

Blood flow estimation assumes laminar flow for the entire fractal vessel tree. In large arteries systolic aberrations of laminar flow is a result of wave propagation. Turbulent flow is also observed in pathological vessels. In small arteries, which are subject to described research, assumption of laminar flow is correct. Hydrodynamic equations for small arteries give correct results in biological circulatory system [4–6,16,24,25].

3. Poiseuille's law

Consider a steady; laminar flow of Newtonian fluid with constant viscosity through a horizontal cylindrical rigid tube (vessel segment). For such model Navier–Stokes equations are simplified to Poiseuille's equation.

$$Q = \frac{\pi * r^4}{8\mu} \frac{\Delta p}{L} \tag{1}$$

Poiseuille's law relates the blood flow Q [ml/s] through a blood vessel with the difference in blood pressure at the two ends of vessel segment Δp created by the heartbeat, radius r, length L, and viscosity μ of the blood, which correlates to hematocrit.

The most effective factor controlling blood flow is radius of the blood vessel. High blood pressure can be caused by narrowing blood vessel and is reduced by relaxing the smooth muscle tension that controls the blood vessel radius. This process is known as an autoregulation.

During laminar blood flow cylindrical layers of liquid are exposed to internal friction, which represents resistance of flow R.

$$R = \frac{8\mu L}{\pi * r^4} \tag{2}$$

The resistance of blood motion through vessels is most strongly dependent on radius, with the fourth power relationship.

Because of fluid friction blood flow velocity within vessel varies from none in wall proximity to maximum value in the center of the vessel creating parabolic velocity profile. Average velocity (with respect to cross-section) inside blood vessel segment is determined as:

$$U_m = \frac{\Delta p * r^2}{8\mu * L} \tag{3}$$

Mathematical models of blood flow in small arteries are usually based on Poiseuille's equation [9]. These equations give acceptable results in biological circulatory system [4,13,15,25].

4. Mass conservation

Compliance with mass conservation law requires that flow rate in parent vessel must equal the total flow rate in children vessels on each branching level. Assuming that flow rate is constant for individual blood vessel the continuity equation is expressed by

$$Q_0 = Q_1 + Q_2 \tag{4}$$

At an arterial bifurcation the flow in the parent vessel Q_0 equals the sum of flow in two children segments Q_1 and Q_2 respectively.

5. Boundary conditions

Flow evaluation starts at terminal vessel and proceed to the root segment. With each terminal vessel equal flow value is associated. Predefined flow is typical for size bracket terminal branches belong to [14,20,21]. Recurrent procedure gives flow parameters evaluation on lower branching level according to mass conservation law and Poiseuille's law.

In order to illustrate the impact of structure and functional characteristic of the arterial tree, simulations have been done for different fractal blood vessel trees with different value of bifurcation exponent. Only asymmetrical trees have been tested according to previously studied impact of tree asymmetry for transport vessels geometrical properties. Symmetrical trees are concerned in capillary bed mainly. In capillary system vessel diameters are similar to the size of blood cells so blood flow considerations based on Poiseuille's law are not suitable here.

Bifurcation exponent determines cross section area of children vessels. After bifurcation in circulatory system total cross section area of children vessel is always bigger than cross section area of parent vessel [25]. So bifurcation exponent must be bigger than 2. In literature the exponent value for human arterial tree is equal 2.6–2.7 [3,11,12,23]. For large arteries where blood flow can be turbulent the bifurcation exponent should be around 2.33 [26]. For capillaries is most likely to be near 3 [3].

Usually in flow analysis vessel symmetry is assumed mainly for sake of simplicity. Arterial vessels split themselves asymmetrically mostly. Capillaries forks symmetrically are exception to the rule.

6. Flow analysis

Hydrodynamical parameters have been tested for different bifurcation exponent values. Bifurcation exponent, one of the most important parameters in transport vessel tree, essentially determines flow through circulatory system.

Pressure drop is monotonically descending positive function. Expected asymptotes for bifurcation exponent $n = 2.5$ is close to 0. Computation performed for higher vessels diameters, for diameter over 1000 μm pressure drop reaches 0.08 kPa, proves descending tendency visualized in Fig. 1. Typical blood vessel system which can be best described by bifurcation exponent $n = 2.7$ represents similar trend, although asymptote is expected to be on higher level then for $n = 2.5$. Asymptote for $n = 3$ is approximately 0.88 kPa (Fig. 2).

Vessel termination ratio represents percentage of vessels which are terminated on defined level divided by all vessels on this level. For subsequent bifurcations flaw rate falls monotonically. Irregular curves after 13th bifurcation occurs

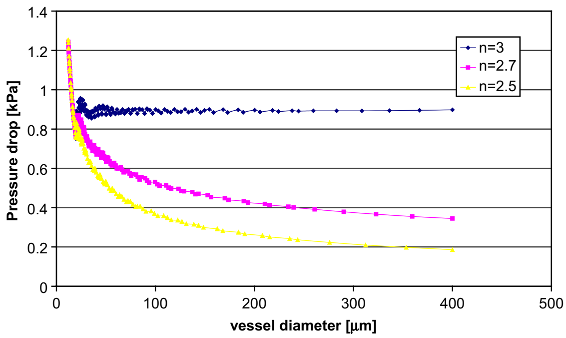

Fig. 1. Pressure drop for different vessel diameters.

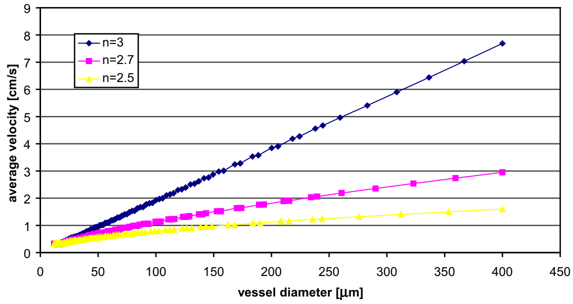

Fig. 2. Average velocity for different vessel diameters.

due to finished terminal vessels, which are not taken into volumetric rate aggregation. Percent of finished vessels on defined level precedes abrupt volumetric flow increase. Logarithmic OY scale emphasis this feature. Every physical structure, which is finite, would represent similar volumetric flow scattering for higher bifurcation levels. Obtaining asymptotic solution for higher bifurcation levels requires more efficient computational resources (Figs. 3 and 4).

Flow heterogeneity is a necessary consequence of a uniform shear stress distribution. Both homogeneous perfusion and uniform shear stress are desirable goals in real arterial trees but each of these goals can only be approached at the expense of the other. Relation between the heterogeneities in flow and shears stress may represent a more general principle of vascular system [22,30–33].

The ability to predict blood flow along arteries can lead to a better understanding of the arterial function.

7. Correction to Poiseuille's law

Parabolic velocity profile during laminar blood flow in individual vessel is disturbed during vessel bifurcation. Development of parabolic flow profile after bifurcation requires correction to Poiseuille's law. Difference in blood pressure at the two ends of vessel is express by

$$\Delta p = A_1 Q + B_1 Q^2 \tag{5}$$

This relationship is known as *Forchheimer equation*.

The first element describes pressure drop during parabolic velocity profile along individual vessel. Coefficient value, A_1 equals the resistance to flow according to Poiseuille's law $A_1 = R$. The second square element describes additional pressure drop as a cause of shape parabolic flow profile. Value B_1 is expressed by

$$B_1 = \rho \frac{\beta}{\pi r^4} \tag{6}$$

$\beta \approx 0.639$ average value from Navier–Stokes equation, ρ blood density [4].

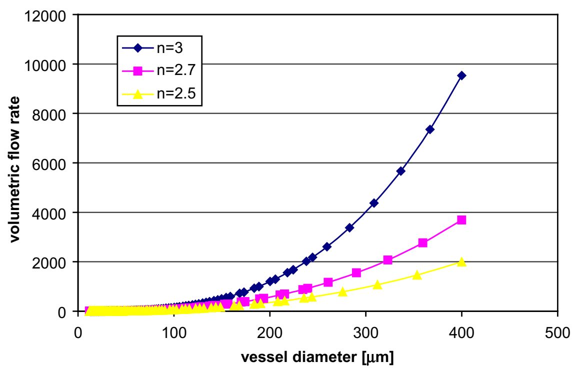

Fig. 3. Volumetric flow rate for different vessel diameters.

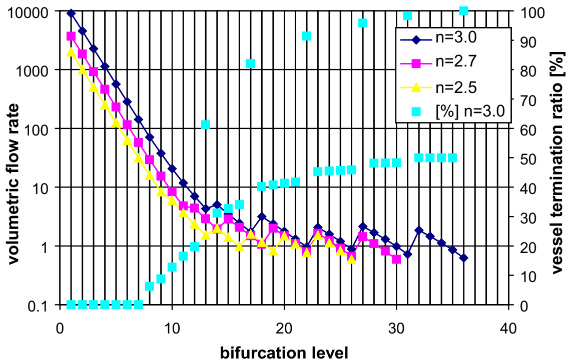

Fig. 4. Volumetric flow rate for different bifurcation levels and vessel termination ratio for selected bifurcation exponent.

Pressure drop have been calculated according to Forchheimer equation. Because of very small value of flow rate [10^{-6} cm^3/s] for tested fractal vessels, the essential differences between Poiseuille's Law and Forchheimer equation are neglectable.

Parabolic velocity profile formation, after bifurcation, is concerned on vessel inlet section l_e. The length of this section is proportional to vessel diameter, Reynolds number and coefficient λ

$$l_e = \lambda dRe \tag{7}$$

$$Re = \frac{V * l}{\mu} \tag{8}$$

$\lambda = 0.056$ a value obtains after numerical calculation of Navier–Stokes equation (4).

Vessel nodes haven't as big influence on flow as shaping parabolic velocity profile. In researches, nodes influences have been omitted.

8. Simulation results

Length of parabolic velocity profile formation (inlet section l_e) has been evaluated. Influence of bifurcation exponent value on inlet section l_e for blood vessel fractal models has been explored.

The length of inlet section depends on bifurcation exponent and Reynolds number mainly. Speed reduction, coherent with lower bifurcation exponent n, is a predominant factor of inlet section length reduction. If inlet section is longer or equal vessel length than asymmetrical velocity profile follows to the next generation of vessels. The process also involves superposition with inlet section of proceeding vessel [27–29] (Fig. 5–7).

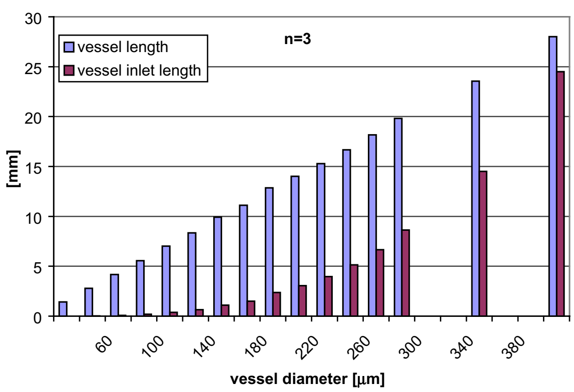

Fig. 5. Length of vessel inlet section for bifurcation exponent $n = 3$.

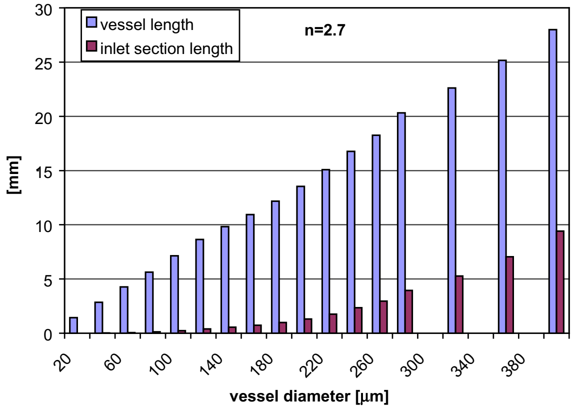

Fig. 6. Length of vessel inlet section for bifurcation exponent $n = 2.7$.

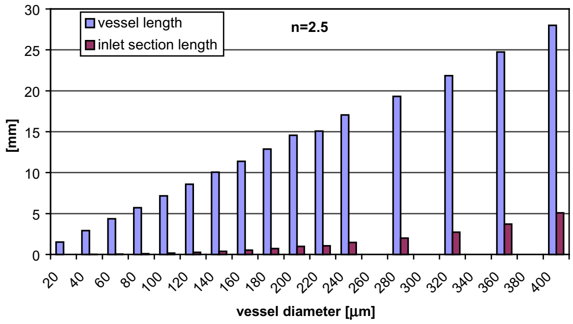

Fig. 7. Length of vessel inlet section for bifurcation exponent $n = 2.5$.

Due to relatively low flow velocity nonlinearities are not common in vessels of small diameter. This conclusion opposes suggestion of predominant nonlinear flow in small arteries proposed in [4]. Provided Reynolds' number remains small along with reducing diameter the inlet section length is also reduced. Reduction is not only in absolute units but relative to vessel length as well.

9. Conclusion

Proposed models of fractal arterial trees have been scaled in such a way that the principal statistical characteristics are quantitatively comparable with morphometry measurements of real arterial trees. Flow model for defined asymmetrical vascular tree has been proposed. Possible simplification and generalizations assumed for aforementioned model have been discussed. Proposed approach enables straightforward analytical evaluation of flow parameters, which usually involves very sophisticated numerical computational methods like FEM. Especially for such complicated phenomena as vascular structures and circulation in general.

Irregularities which have been observed during volumetric flow rate are possible for computational model only. In real situation circulatory system is a close system where after terminal small arteries, capillaries vessel are appeared. Because of nearly the same size of blood cells and vessel diameter hemodynamical equations could not be applied on capillaries level.

Terminal vessels flow is equal for all vessel trees. Blood flow in the other vessels depends on bifurcation quantity and bifurcation exponent mostly. Bifurcation exponent decides about vessel length on given level and step number from root vessel to terminal vessels.

References

[1] Ahlqvist J. Atherosclerosis, and Newton, Poiseuille, Reynolds and Prandtl. Med Hypotheses 2001;57(4):446–52.
[2] Boryczko K. Computational model of blood dynamics in capillary vessels. Doctoral thesis, 2003.
[3] Changizi MA, Cherniak C. Modeling the large- scale geometry of human coronary arteries. Can J Physiol Pharmacol 2000;78:603–11.
[4] Cieślicki K. Hydrodynamiczne uwarunkowania kra żenia mózgowego. AOW EXIT, Warszawa, 2001.
[5] Cohn D. Optimal systems I: the vascular system. Bull Math Biophys 1954;16:59–74.
[6] Cohn D. Optimal systems II: the vascular system. Bull Math Biophys 1955;17:219–27.
[7] Fernandez MA. Analysis of a geometrical multiscale blood flow model based on the coupling of ODE's and hyperbolic PDE's. Inria-Bang, 2004.
[9] Iordache BE, Remuzzi A. Numerical analysis of blood flow in reconstructed glomerular capillary segments. Microvasc Res 1995;49:1–11.
[10] Kędzia A, Rybaczuk M, Dymecki J. The fractal analysis of subdural haematoma. Folia Neuropathol 39(3):203–207.
[11] Kurz H, Sandau KS. Modeling of blood vessel development—bifurcation pattern and hemodynamics, optimality and allometry. Comments Theor Biol 1997;4:261–81.
[12] Mandelbrot B. The fractal geometry of nature. Freeman; 1977.
[13] Nonnenmacher TF. Fractal in biology and medicine, 1993.
[14] Pilawski A. Podstawy biofizyki. Podre dla studentów medycyny, 1985.
[15] Pries AR, Secomb TW, Gaehtgens P. Biophysical aspects of blood flow in microvasculature. Cardiovasc Res 1996;32:654–67.
[16] Rosen R. Optimal principles in biology. London: Butterworths; 1967.
[17] Rybaczuk M, Kędzia A, Błaszczyk E. Fractal description of cerebellum surface during fetal period. Folia Morphol 1996;55:434–6.
[18] Rybaczuk M, Kędzia A, Andrzejak R. Fractal dimension of human brain cortex vessel during the fetal period. Med Sci Monit 2002;8(3).
[19] Rybaczuk M, Kędzia A, Paradowski L. Fractal characteristic of brain vessel microangioarchitecture during the fetal period. Med Sci Monit 2002;8(8):MT145–52.
[20] Schreiner W. Structural quantification and bifurcation symmetry in arterial tree models generated by constrained constructive optimization. J Theor Biol 1996;180:161–74.
[21] Schreiner W, Neumann F, Karch R. Shear stress distribution in arterial tree models generated by Constrained Constructive Optimization. J Theor Biol 1999;198:27–45.
[22] Schreiner W, Karch R. Heterogeneous perfusion is a consequence of uniform shear stress in optimized arterial tree models. J Theor Biol 2003;220:285–301.
[23] Suwa N, Fukasawa H, Sasaki Y. Estimation of intravascular blood pressure gradient by mathematical analysis of arterial casts. Tohoku J Exp Med 1963;79:168–98.
[24] Suwa N, Takahashi T. Morphological and morphometrical analysis of circulation in hypertension and ischemic kidney. Urban & Schwarzenberg, 1971.
[25] Traczyk W, Trzebski A. Fizjologia człowieka z elementami fizjologii stosowanej i klinicznej. PZWL, 2001.
[26] Uylings H. Optimization of diameters and bifurcation angles in lung and vascular tree structures. Bull Math Biol 1977;39:509–20.
[27] Weinbaum S, Jiji LM. Theory and experiment for the effect of vascular microstructure on surface tissue heat transfer—Part I: Anatomical foundation and model conceptualization. Trans ASME J Biom Eng 1984;106:321–30.
[28] West GB, Brown JH, Enquist BJ. A general model for the origin of allometric scaling laws in biology. Science 1997;276:122–6.
[29] Zamir M. Optimal principles in arterial branching. J Theor Biol 1976;62:227–51.
[30] Zamir M. The role of shear forces in arterial branching. J Gen Physiol 1976;67:213–22.
[31] Zamir M, Braun N. Arterial branching in various parts of the cardiovascular system. Am J Anat 1982;163:295–307.
[32] Zamir M. The branching structure of arterial trees. Comments Theor Biol 1988;91:15–37.
[33] Zamir M. Distributing and delivering vessels of the human heart. J Gen Physiol 1988;91:725–35.