

PHOTONIC FRACTALS OF DIELECTRIC AND METAL MEDIA
FOR ELECTROMAGNETIC WAVE LOCALIZATION

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ABSTRACT

3D photonic fractals with the self-similar structure can localize electromagnetic waves completely without reflection and transmission. Such fractals of the Menger sponge structure were successfully fabricated with dielectric and metal media using CAD/CAM stereolithography. The localized frequency in dielectric Menger sponge structures is predicted using an empirical equation associated with the fractal geometry and spatially averaged dielectric constant, thus the frequency of the localized modes can be designed. For metal Menger sponge structures, the localized frequency could be determined mainly based on the geometrical factors. The localization mechanism in a photonic fractal is essentially different from the localization in a photonic crystal with the periodic structure. These new functionally structured materials of photonic fractals are considered to be applicable in a wide area of communication, information, energy, sensing, medical care, and others.

INTRODUCTION

Electromagnetic or optical wave interactions with periodic structures like photonic crystals have been studied intensively in the last two decades. Bragg scattering in photonic crystals creates the photonic band gap and localization of electromagnetic wave.¹⁻³ It is known that the band gap is formed not only in dielectric photonic crystals, but also in metallic ones.⁴⁻⁵

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While, the wave interactions with fractal structures have attracted attention in recent years as well. Fractal structure is characterized with the self-similarity which means that the local configuration is similar to the whole configuration.⁶ It has no periodic symmetry. Such self-similar structures show interesting reflection and transmission phenomena.⁷⁻⁹, and strong localization of electromagnetic or optical waves.¹⁰ However, these studies were limited in fractal planes or multilayers with 1 or 2 dimensional geometry so that the complete localization in a three dimensional space has not been achieved.

Recently, we have fabricated 3D fractal structures called Menger sponge with epoxy or TiO₂-SiO₂ dispersed epoxy by CAD/CAM stereolithography and found that the incident microwave with the specific wavelength is completely localized or confined without reflection and transmission.¹¹⁻¹² Because no such material was known that can localize electromagnetic waves three dimensionally without reflection and transmission, we named it as photonic fractal. Furthermore, we have found that metal Menger sponge structures can also localize electromagnetic waves. Such localization function of electromagnetic waves or energy is applicable in communication, information, energy, sensing, medical care, and other fields.

In the present paper we report on fabrication of such 3D fractal structures of dielectric and metal media, their electromagnetic wave responses, and examine in some detail the localization behaviors.

GEOMETRY OF MENGAER SPONGE FRACTAL

Menger sponge is a 3D version of the Cantor bar fractal. Cantor bar is formed by extracting a center segment of three equivalent segments divided from an initial bar and repeating this process to the remaining two side segments. When this operation is conducted one time, the resulting Cantor bar is called stage 1. By repeating the operation, the stage number increases as shown in Fig.1 (a). Figure 1 (b) is called Sierpinski's carpet which is a two dimensional version of Cantor bar. Its stage number is 3. Figure 1 (c) is a stage 3 Menger sponge, which is formed by dividing the initial cube into 27 identical cube pieces, and extracting seven pieces at the body- and face-centers.

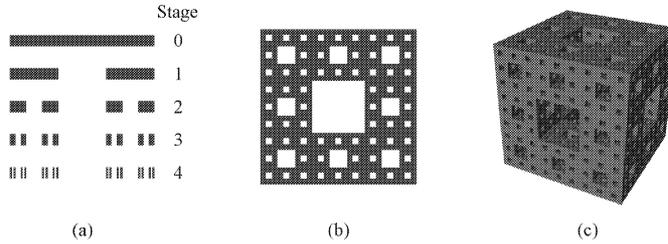


Figure 1. 1D, 2D, and 3D fractals; (a) Cantor bar, (b) Sierpinski's Carpet, (c) Menger sponge.

The fractal dimension D is used as an important parameter to describe the complexity of fractal structures, which is calculated by the following equation:

$$N=S^D \quad (1)$$

where N is the number of the self-similar units newly created when the size of the initial unit decreases to $1/S$. In Cantor bar, $N=2$ and $S=3$, so that $D=\log 2 / \log 3=0.63$, in Sierpinski's Carpet, $N=8$, $S=3$, $D=\log 8 / \log 3=1.89$, in Menger sponge, $N=20$, and $S=3$, $D=\log 20 / \log 3=2.73$. It is noted that the fractal structure is characterized mathematically by the fractal dimension with decimal figures.

EXPERIMENTAL

We designed dielectric Menger sponge structures on a computer using a CAD program (Toyota Caelum Co. Ltd, thinkdesing ver.5.0). The designed structure is converted into a rapid prototyping format (STL file), sliced into a set of thin sections, and transferred to a stereographic machine (D-MEC Co.Ltd, Japan, SCS-300P). This machine forms a three-dimensional object layer-by-layer by scanning a UV laser of 355 nm in wavelength over a liquid photopolymer epoxy resin.¹³ The thickness of each layer is about $150 \mu\text{m}$. The $\text{TiO}_2\text{-SiO}_2$ powders with a particle size of about $10 \mu\text{m}$ are dispersed into the liquid resin in order to increase the dielectric constant of the fractal structure.

Metallic Menger sponge structures were fabricated by vacuum casting the molten Bi-Pb-Sn-In eutectic alloy with melting point at 70°C into the epoxy mold, which was

for the stage 4 Menger sponge with the same size as the stage 3 Menger sponge.

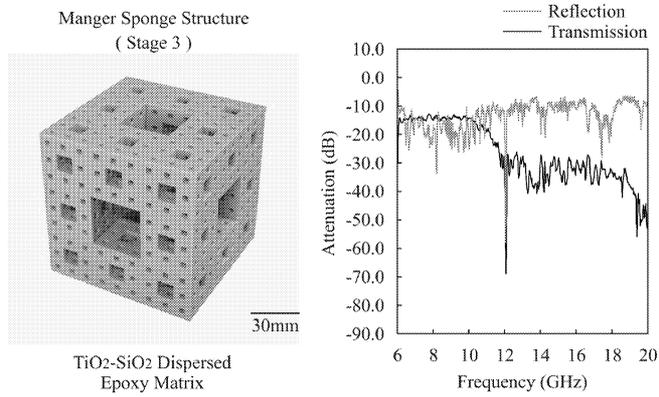


Figure 3. Photo of a stage 3 Menger sponge structure of $\text{TiO}_2\text{-SiO}_2/\text{epoxy}$ with 81mm cube, and the reflection and transmission spectra.

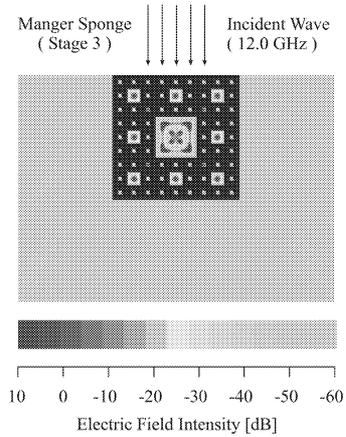


Figure 4. Profile of electric field intensity at the cross sectional x-y plane in a stage 3 Menger sponge structure of $\text{TiO}_2\text{-SiO}_2/\text{epoxy}$ with 81mm cube.

However, the electric field leaked out of the stage 2 Menger sponge though the resonated intensity was observed in the air cubes as seen in Fig.5. Figure 6 shows a

stage 4 Menger sponge with an edge length of 81mm and the transmission and reflection spectra. Sharp attenuations for both reflection and transmission to -70dB were observed at 13.6 GHz. These results suggest that the strong localization of microwaves occurs in dielectric Menger sponge structures with the stage numbers higher than 3.

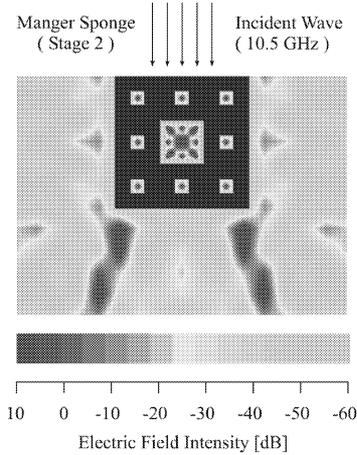


Figure 5. Profile of electric field intensity at the cross sectional x-y plane in a stage 2 Menger sponge structure of $\text{TiO}_2\text{-SiO}_2/\text{epoxy}$ with 81mm cube.

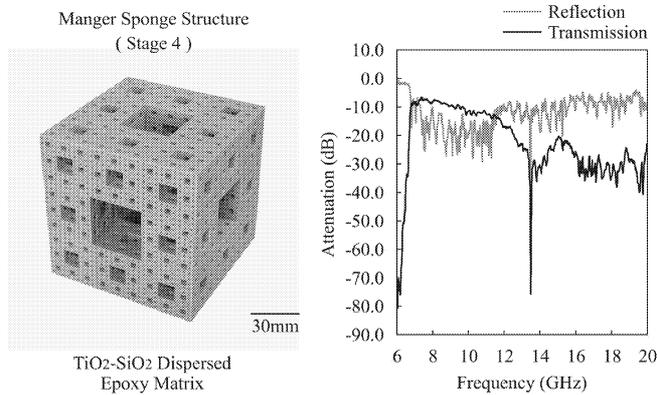


Figure 6. Photo of a stage 4 Menger sponge structure of $\text{TiO}_2\text{-SiO}_2/\text{epoxy}$ with 81mm

cube, and the reflection and transmission spectra.

The localization of electromagnetic waves in dielectric Menger sponges can not be explained by Bragg scattering and the band gap produced in periodic structures like photonic crystals. Probably such localization would occur due to the self-similar resonances of multiply reflected or scatted waves in a 3D fractal structure. Though the localization mechanism of electromagnetic waves in these Menger sponge fractals is not well understood yet, an empirical equation could be derived to predict the frequency of the localized mode in dielectric Menger sponges as follows.

$$\lambda_{\text{conf}} = 2^{\ell} a \sqrt{\epsilon_{\text{eff}}} / 3^{2\ell-1} \quad (2)$$

where λ_{conf} is the wavelength of the localized mode in air. ℓ is the order number of the localized modes. The effective dielectric constant ϵ_{eff} is the spatially averaged dielectric constant of the Menger sponge structure. The filling factor f is the volumetric ratio of remaining parts in an n-dimensional fractal. In the case of the stage m Menger sponge, it is expressed by $f = (N/S)^m$. Thus, the filling factor of dielectric medium in the stage 3 Menger sponge is $(20/27)^3$. The effective dielectric constant ϵ_{eff} is given by

the equation of $\epsilon_{\text{eff}} = f\epsilon_A + (1-f)\epsilon_B$, where ϵ_A and ϵ_B are dielectric constants of remaining and extracted parts, respectively. The dielectric constant of the bulk epoxy including 10vol.%TiO₂-SiO₂ measured was $\epsilon_A=8.8$. $\epsilon_B = 1$ (air). Therefore, the effective dielectric constant for the stage 3 Menger sponge is calculated as 4.2.

Table I compares the calculated and measured frequencies of localization in the dielectric Menger sponges with different sizes, stages, and dielectric constants. Other data for stage 3 Menger sponges (27mm cube) of epoxy and TiO₂-SiO₂/epoxy, which were obtained in our previous study, are added in the table.¹¹⁻¹² In these smaller Menger sponges, the localized frequencies measured at 12.8GHz for the epoxy and 8.0 GHz for the TiO₂-SiO₂/epoxy show good agreements with the calculated ones as the first localized mode ($\ell=1$).

Table I. Measured and calculated frequencies of localized modes in various Menger sponge structures.

Cube Size; a [mm]	Stage Number; m	Effective Dielectric Constant; ϵ_{eff}	Calculated Localized Frequency ($\ell=1$) [GHz]	Calculated Localized Frequency ($\ell=2$) [GHz]	Measured Localized Frequency [GHz]
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27	3	1.7	12.8	57.5	12.8 ¹¹
27	3	4.2	8.1	36.6	8.0 ¹²
81	3	4.2	2.7	12.2	12.0
81	4	3.3	3.1	13.8	13.6

The calculated frequencies for the second modes ($\ell=2$) were 57.5 GHz and 36.6 GHz, respectively. These frequencies were high and out of measurable range in our measurement system.

For the present stage 3 and 4 Menger sponges (81mm cube), the measured frequencies were 12.0 GHz and 13.6 GHz, respectively. These values agree well with the calculated ones for the second modes. The calculated frequencies for the first modes were out of measurable range.

In a dielectric medium with the dielectric constant ϵ , the wavelength of electromagnetic wave is reduced to $1/\sqrt{\epsilon}$. Equation (2) reveals that the wavelength of the localized mode in a dielectric Menger sponge structure corresponds to 2/3 of its edge length for the first mode and 4/9 for the second mode. This means that the strong localization can occur in such a narrow space with the size of 1.5 or 2 times of the wavelength in dielectric Menger sponges with the stage number greater than 3. It is very efficient comparing with photonic crystals, which need a space of periodic structure with several times of wavelength for the strong localization.

Localization in Metal Menger Sponge Fractals

Figure 7 shows a stage 3 metal Menger sponge fractal with an edge length of 81 mm and the transmission and reflection spectra against normally incident microwaves. The metal Menger sponge was well fabricated without distortions, cracks, and pores. Very sharp attenuations of transmission and reflection down to -80 dB appeared at 15.1 GHz. The strong intensity profile of the electric field at 15.1 GHz was observed in the central air cube as shown in Fig.8. Stage 2 and 4 metal Menger sponges with the same size as the stage 3 showed more shallow and deep dips without leaking of incident microwave at the same 15.1 GHz, respectively, however, the stage 1 sample showed no localization.

P. Sheng and his co-workers reported on anomalous behaviors in reflectance and transmittance of microwaves for two dimensional Cu metal fractals.⁸⁻⁹ While, Shalaef's group found extremely strong localizations of light at random fractal films consisting of nano-sized silver particles aggregate.¹⁰ However, in these 2D fractal structures, it is impossible to completely localize the incident microwaves or light.

In the case of 3D dielectric Menger sponge fractals, the localized wavelength and frequency depended on the effective dielectric constant, which changes with the stage number even though the size and material are the same. While the metal Menger sponges showed no dependence of the localized frequency on the stage numbers. The dielectric constant of metal is negative so that electromagnetic waves can not penetrate into metal and scatter at the surface. Figure 7 shows the existence of the cut-off frequency at about 6 GHz. It corresponds to the wavelength of 50mm. The edge length of the largest central air cube is 27mm near the half wavelength, while, the wavelength of the localized mode at 15.1 GHz is about 20 mm.

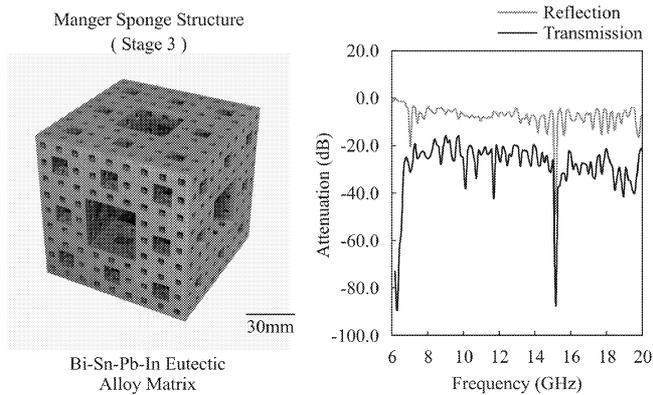


Figure 7. Photo of a stage 3 Menger sponge structure of Bi-Sn-Pb-In eutectic alloy with 81mm cube, and the reflection and transmission spectra.

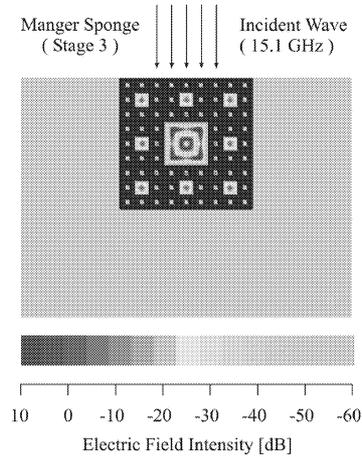


Figure 8. Profile of electric field intensity at the cross sectional x-y plane in a stage 3 Menger sponge structure of Bi-Sn-Pb-In eutectic alloy with 81 mm cube.

The $3/2$ and $1/2$ wavelengths correspond nearly to each edge length of the center (27mm) and second air cubes (9mm), respectively. It is considered, therefore, that the measured localized mode may exist mainly in the central and second air cubes. The electric field intensity profile measured supports this estimation.

At present, we can not explain why electromagnetic waves localize in these dielectric and metal Menger sponge fractals, but the 3D fractal structure should play an essential role for the complete localization of waves. The theoretical study on the localization mechanism will be reported elsewhere.¹⁴

POTENTIAL APPLICATIONS OF PHOTONIC FRACTAL

The localization function of electromagnetic waves in 3D dielectric or metal fractal structures is applicable to various fields of information, communication, energy, medical care, and others. Fractal structures such as Menger sponge can provide an ideal absorber without reflectance and transmission for electromagnetic waves. Of course, when the dielectric fractal structure is made in optical wave scales, the light can be localized and confined that will be used for many photonic applications. It may be possible to localize and accumulate the electromagnetic energy in dielectric or metal fractal structures, if the structure is composed of an efficiently low loss material and the number of stage m is sufficient which can reduce the medium of fractal. Such energy

accumulation in the 3D fractal structure may provide new heat treatment devices for industrial, home and medical uses depending on the power. If high efficiency in energy accumulation could be realized by choosing ceramic, metal or their composite materials, new functional devices of photonic energy storage, capacitor, and collector may be produced.

CONCLUSION

3D dielectric and metal photonic fractals of Menger sponge structure were successfully fabricated by using CAD/CAM stereolithography and their electromagnetic responses were measured. The obtained results can be summarized as follows.

1. Dielectric and metal Menger sponge structures with the stage number over 3 can localize strongly electromagnetic waves with the specific wavelength.
2. The wavelength and frequency of the localized modes in dielectric Menger sponges can be predicted using an empirical equation associated with the fractal geometry and spatially averaged dielectric constant, thus the frequency of the localized modes can be designed. For metal Menger sponges, the localized frequency may be determined mainly by the geometrical factors.
3. The localization mechanism in these dielectric and metal fractals is not well understood yet, however, the 3D fractal structure of Menger sponge would play an essential role for the localization.
4. Photonic fractals exhibit great potential for applications in information, communication, energy, medical and other various fields.

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