Toolpath generation for layer manufacturing of fractal objects

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Abstract

Purpose – Fractal geometry can be used to model natural objects which cannot be easily represented by the euclidean geometry. However, contemporary computer-aided design (CAD) and computer-aided manufacturing (CAM) systems cannot be used to model a fractal object efficiently. In a general layer manufacturing (LM) workflow, a model described by the euclidean geometry is required in order to generate the necessary toolpath information. So this workflow cannot be applied for a fractal object. In this paper, to realize the fabrication of a fractal represented object by the LM technology, a methodology is proposed.

Design/methodology/approach – In the proposed methodology, a slab grid is generated in each layer of the object and it consists of a number of pixels. The interior property (corresponding to the fractal object) of each pixel in the slab grid is checked so that slab models of the fractal are created. The boundary of each slab is traced and refined so that the toolpath of the object can be generated from these boundaries.

Findings – Applying the proposed methodology, the LM toolpath information can be extracted from the mathematical model of the fractal and the tessellating or slicing processes are not needed to be performed. The problem of representing a fractal in a CAD platform can be eliminated.

Research limitations/implications – The proposed methodology can be applied to iterative function system (IFS) or complex fractal. However, for some fractals constructed from more than one kind of fractal objects, such as multi-IFS fractals, the methodology must be further developed.

Originality/value – The proposed methodology is a novel development for realizing the fabrication of fractal objects by the LM technology.

Keywords Computer aided design, Computational geometry, Modelling

Paper type Research paper

1. Introduction

1.1 Contemporary CAD/CAM

Contemporary computer-aided design (CAD)/computer-aided manufacturing (CAM) theories and systems are well developed only for euclidean analytical objects and free-form objects (e.g. car bodies). Euclidean geometry (Figure 1) is comprised of lines, planes, rectangular volumes, etc. Some of the free-form curve representations used in contemporary CAD systems are Bezier, B-spline and the most general non-uniform rational B-spline (NURBS) curves. They present usually smoother shapes and are indicated for modeling organic shapes. Euclidean geometries are defined by algebraic formula, for example, $x^2 + y^2 = r^2$ is used to define a sphere. These elements can be classified as belonging to an integer dimension either one, two, or three. This concept of dimension can be described both intuitively and mathematically (Mortenson, 1995).

Currently, most of the commercial CAD systems are either based on solid or surface modeling systems. Two common choices are usually used to represent a 3D object, a boundary based representation called boundary representation (B-Rep), and a volume-based representation, called constructive solid geometry (CSG). Solid modeling, usually encountered as CSG, is used to create complex solid models by combining simple solid primitive. CSG has its mathematical foundations in topology, algebraic geometry, and Boolean algebra. However, depending on a particular CAD modeling system being used, the modeling capability is limited by the availability of solid primitives. Hence, B-Rep distinguishes vertices, edges, and polygons, but no explicit relations are maintained between them.

However, there are still many types of objects, such as flexible objects with deformable geometry and non-euclidean geometrical objects, that cannot be represented efficiently by the current available CAD representation schemes. For some fragmented and self-similar aesthetic products, such as the example shown in Figure 2, geometric tools other than the euclidean one are required. In this case, the fractal geometric theory can be used to mathematically define these types of products. However, these fractal objects are usually used as textures in the field of computer graphics and can only be visualized via the computer graphics systems. In these cases, the fractals are stored as an array of pixel elements. At present, no representation schemes are available for storing the geometrical and topological information of a fractal model and processing of this fractal object is thus difficult.

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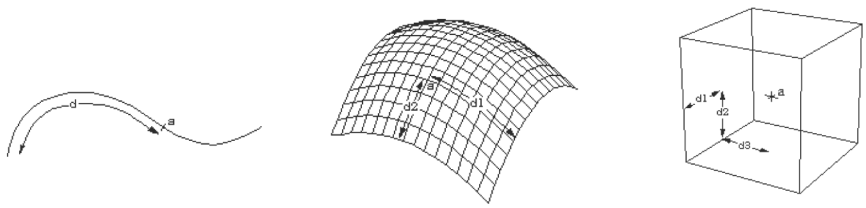
Figure 1 Euclidean geometry

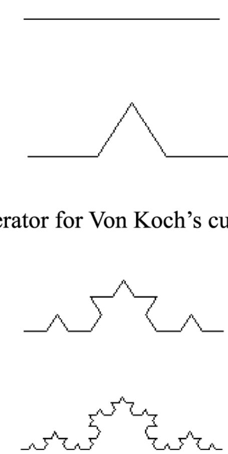


(a) A curve **(b)** A surface **(c)** A volume

Figure 2

Source: http://www.fractalus.com/sharon/xeno/xeno.htm

Figure 3 Recursive process of Koch curve



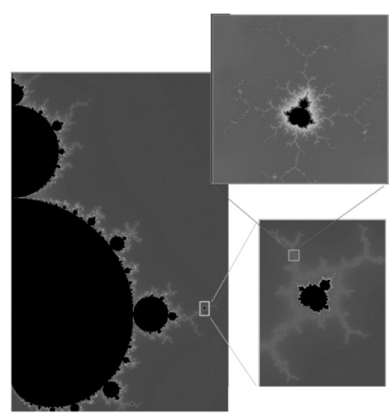
1.2 Fractal object

The term "fractal" was introduced for characterizing spatial or temporal phenomena that are continuous but not differentiable (Mandelbrot, 1975). Fractal is defined as a rough or fragmented geometric shape that can be sub-divided into parts, each of which is (at least approximately) a reduced-size copy of the whole. Mathematically, a fractal is defined as a set of points whose fractal dimension exceeds its topological dimension (Mandelbrot, 1983). In general, the dimension of a fractal is typically a non-integer or a fraction, meaning its dimension is not a whole number and its formation is by an iteration (or recursive) process (Figure 3), and hence has non-integer complexity. When a fractal is magnified, it is infinitely complex (Figure 4). Moreover, upon magnification of a fractal, it can be found that subsets of the fractal resemble the whole fractal, i.e. self-similar (Figure 5).

There are two types of fractal geometries – iterative function system (IFS) fractal and complex fractal. The classification of these two types of fractals is shown in Figure 6. In general, an IFS fractal is a family of specified mappings that map the whole onto the parts and the iteration of these mapping will result in convergence to an invariant set. There are numerous literatures about IFS and readers can find the details from them (Moran, 1946; Williams, 1971; Hutchinson, 1981; Barnsley and Demko, 1985; Barnsley, 1988).

For complex fractal, Julia set and Mandelbrot set are mainly included. The Mandelbrot fractal is generated by iterating a quadratic polynomial in the complex plane f: $\mathbb{C} \rightarrow \mathbb{C}$ and represented in computer graphics (Rojas, 1991). The function

Figure 4 Fractal magnified infinitely



of the Mandelbrot set is defined in the iterative synthetical formulation and the complex plane:

$$M = \{c | z_n \nrightarrow \infty, z_{n-1}^2 + c, z_0 = 0\}$$

where c is a constant and n is the level of iteration.

From classical theorems of Fatou and Julia, associated with each parameter value c, there is a Julia set $\mathcal{J}_c$ which is defined as the boundary of Mandelbrot set:

$$\mathcal{J}_c = \{z | z_n \nrightarrow \infty, z_{n-1}^2 + c, z_0 = z\}$$

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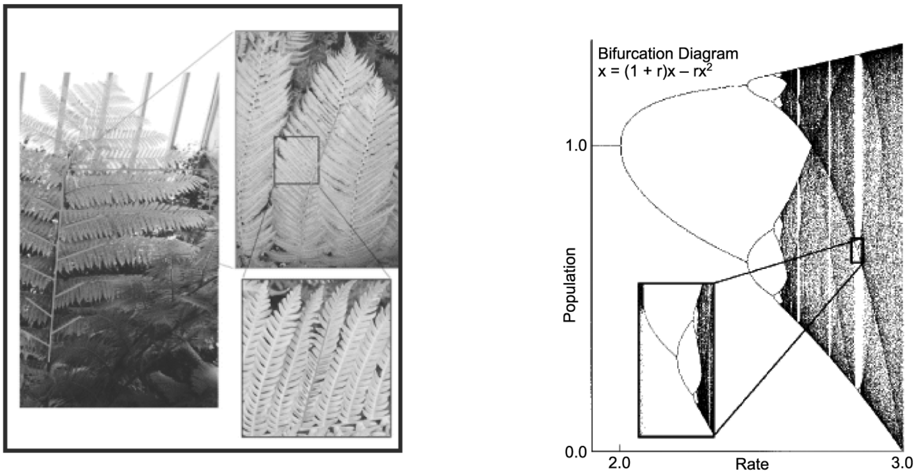
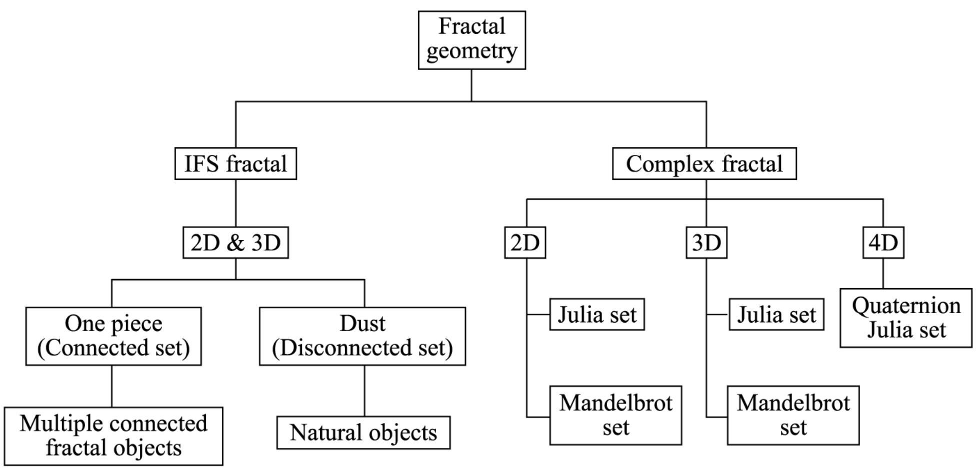
Figure 5



(a) Natural object

(b) The logistic equation

Figure 6 Classification of IFS and complex fractals



where $\mathcal{J}_c$ of function f_c: $z \rightarrow z^2$ is connected if $c \in M$, M consists of all complex numbers c for which the sequence $f(z)$, $f(f(z))$, $f(f(f(z)))$, $f(f(f(f(z))))$, ... remains bounded and does not diverge to infinity. In complex fractal, Mandelbrot set includes the family of Julia set including connected sets (Fatou set) and Cantor sets (Fatou dust). Some special cases for Mandelbrot set are shown in Figure 7. The details of complex fractal can be found in different literatures (Branner, 1989; Peitgen and Richter, 1986; Falconer, 2003; Peitgen *et al.*, 1991).

1.3 Prototype making by layer manufacturing technology

When a physical prototype has to be fabricated, geometrical data of the object must be obtained and input to the corresponding prototype-making process, such as the layer manufacturing (LM) technology. To obtain the geometrical information in each layer, the object must be represented by the euclidean geometry. As mentioned above, for a fractal object (no matter IFS or complex), the object is represented by a mathematical model and can be considered as a points set. Theoretically, it is difficult to slice a "mathematical model" or a "points set" and so it is almost impossible to fabricate a fractal prototype by an LM process without any modification of the current LM workflow. To overcome this problem, one of the methods is to introduce a new representation scheme in a CAD platform to model and approximate a fractal object. Another method is to introduce a new LM workflow so that the geometrical information can be directly extracted from the "points set" or the mathematical model of the fractal. In this paper, the latter approach is discussed and a methodology is introduced. Based on the proposed methodology, tessellating and slicing processes are not needed to be performed for getting the boundary information of the fractal. In Section 2, the traditional LM workflow and some of the applications about fractals are reviewed. The proposed methodology is introduced in Section 3. The results will then be presented in the successive section.

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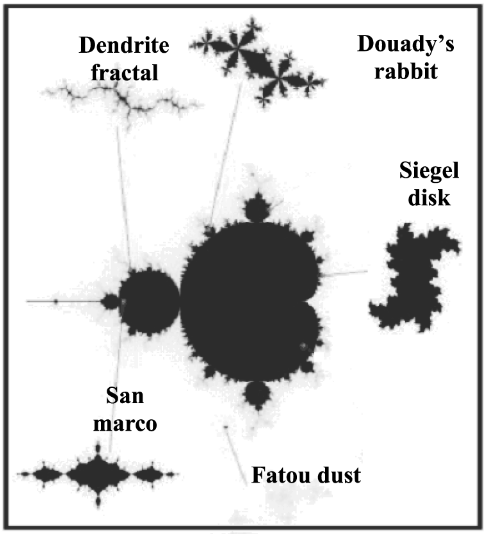
Figure 7 A Mandelbrot set with Julia sets in a constellation diagram



Source: Falconer (2003)

2. Reviews

The traditional workflow for fabricating a prototype from a CAD model by an LM process can be shown in Figure 8. In general, different operations must be performed. For example, the CAD model must be oriented in a proper build direction (some of the works can be found in Bablani and Bagchi (1995), Lan *et al.* (1997), Lin *et al.* (2001) and Xu *et al.* (1999)) when different factors such as build time, surface finish, amount of support are taken into consideration. To obtain the boundary of the model in each layer, a CAD model can be directly sliced (Jamieson and Hacker, 1995) or tessellated into a facet model and sliced (Pandey *et al.*, 2003). Based on the layer boundaries, 2D toolpath for the LM process can be generated. In some researches, one of the fractal curves, the Hilbert curve is used as a new area-filling toolpath generation method for LM processes (Bertoldi *et al.*, 1998) and this method was implemented in Wasser *et al.* (1999).

The Hilbert curve was also used as a toolpath pattern of a robot system for polishing metal mould (Mizugaki and Sakamoto, 1992). Different methods for toolpath generation with the Hilbert curves were also proposed by Zhang *et al.* (2000), Cox *et al.* (1994) and Griffiths (1994). However, these methods are applications of fractal curves only and the prototype-making methods for fractal objects are not discussed.

Apart from the applications of the fractal curves, the fractal growth model has also been applied in a new rapid tooling process, electrochemical liquid deposition based solid free-form fabrication (Zhou *et al.*, 1999). However, this process is not used to fabricate a fractal prototype. For the fabrication of fractal prototypes, Kerekes (1992) discussed this problem and proposed to make a fractal object using the LM technology. Apart from this, Soo and Yu (2001, 2002) also attempted to realize the fabrication of a fractal curve two- and three-dimensionally using the LM technologies, but the making of a 3D fractal object was not mentioned.

3. Fabrication of fractal prototype

3.1 Methodology for generating toolpath information of a fractal object

In the proposed methodology, a fractal object is considered as a stack of 2D slab models and the toolpath information of the object is directly extracted from these. The workflow of the proposed methodology can be shown in Figure 9. The problem of representing a fractal in a CAD platform can thus be eliminated in the fabrication of a fractal by the LM process. The methodology consists of five steps and the details are described blow.

1 *Orienting a fractal object.* The building direction of the fractal prototype must be determined and the fractal object is oriented in the proper direction. Since, the fractal object is not represented in a CAD system, it is impossible to orient the object by some of the optimal build orientation determination methods. In this case, the orientation is defined by a user. Then a group of slicing planes is set such that the distance between successive slicing planes is equal to the layer thickness set in the LM process.
2 *Extracting 2D slab models of the fractal object.* To obtain the necessary layer information, 2D slab model of the fractal object in each slicing plane must be formed. For each slicing plane, a regular 2D slab grid which contains a number of pixels is generated. Each pixel is indexed as (i, j), as shown in Figure 10. The resolution of the slab grid must be set accordingly and this will be discussed below. For each pixel, it will be checked whether it is inside (IN), partially inside (Part-IN) or outside (OUT) the fractal object in a specified resolution. Those IN and Part-IN pixels are kept and used to generate a 2D slab model on the corresponding slicing plane of the fractal object.
3 *Boundary tracing of each 2D slab model.* The Part-IN pixels can be used to define the approximated boundary of the slab model (Figure 11). They are then ordered so that the boundary of the slab model can be obtained. To trace a

Figure 8 Traditional workflow of a LM process

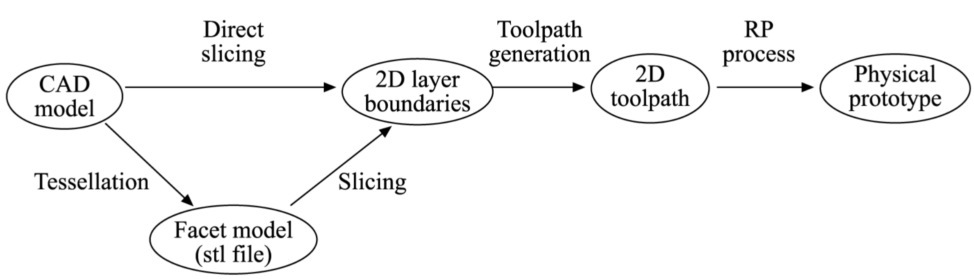


Figure 9 The workflow of the proposed methodology

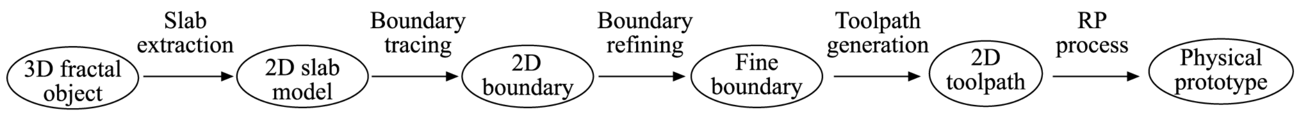


Figure 10 A 2D slab grid

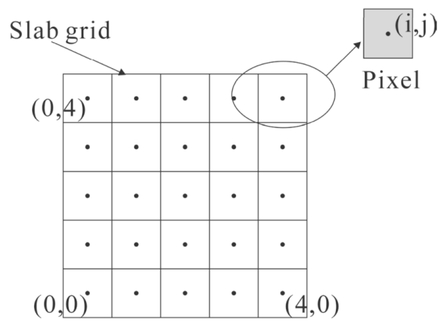
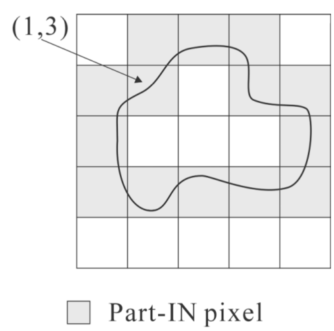


Figure 11 Part-IN pixels of an example 2D boundary



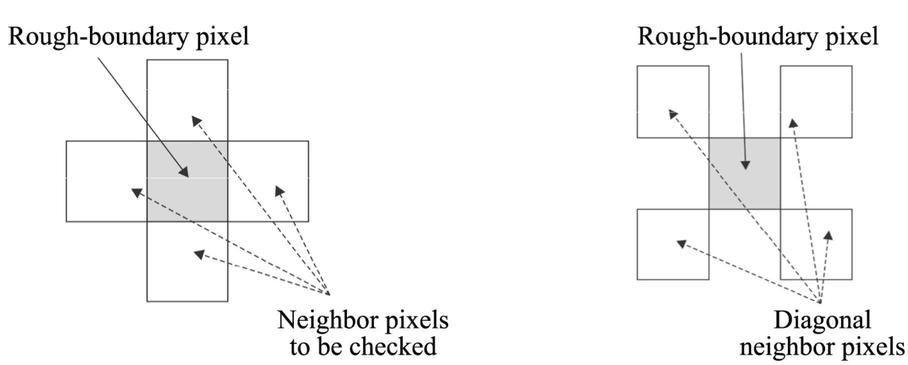
boundary, a pixels-ordering procedure is introduced. A Part-IN pixel is randomly selected as the start of boundary and marked as a rough-boundary pixel. The rough-boundary pixel next to it is then searched. Generally, for each pixel, its four neighbor pixels (Figure 12(a)) are checked and the one which is a Part-IN pixel and not marked is considered as the next rough-boundary pixel. If no Part-IN pixels are found, its four diagonal neighbor pixels (Figure 12(b)) are checked in order to identify the rough-boundary pixel. The procedure is repeated until the whole boundary is traced.

4 *Refining of each Part-IN pixel.* The fineness of the fractal prototype will be affected by the resolution of the slab grid used to generate the slab models. In this step, the level of iteration of the fractal object increases in order to generate a finer fractal object. Then a refining resolution is set. For each Part-IN (or rough-boundary) pixel, the pixel is further sub-divided into sub-pixels by applying the concept of quadtree until the refining resolution is reached. The Part-IN pixel (1,3) in Figure 11 is used as an example and the refining result is shown in Figure 13. Those sub-pixels classified as Part-IN pixels are termed as the boundary pixels. As a result, a finer approximation of the boundary of the 2D slab model can be got.
5 *Generating toolpath.* The boundary pixels obtained in the previous step are ordered by applying the same pixels-ordering procedure described in Step (3) and a boundary toolpath for each layer of the prototype is generated, as shown in Figure 14. The interior areas can be filled by different filling patterns, such as zig-zag pattern or spiral pattern. The complete toolpath is then generated and can be output to an LM process for the fabrication of the fractal prototype.

3.2 Slab grid resolution

As mentioned in the second step, the resolution of the slab grid must be determined and the quality of the slab model would be affected by the fineness of the grid. If a low resolution is set, a low quality boundary would be obtained and more iteration steps would be required in the boundary-refining step. On the other hand, if a high resolution is used, the number of calculation for checking the interior property of each pixel would be numerous. So there must be a trade-off between the resolution and the amount of interior property checking. When a model is fabricated by an LM machine, those features with their sizes smaller that the tool size of the LM machine (e.g. laser Gaussian half-width for SLS process,

Figure 12 Part-IN pixel



(a) Its neighbor pixels **(b)** Its diagonal neighbor pixels

Figure 13 Refining the Part-IN pixel of a 2D slab model by quadtree operation

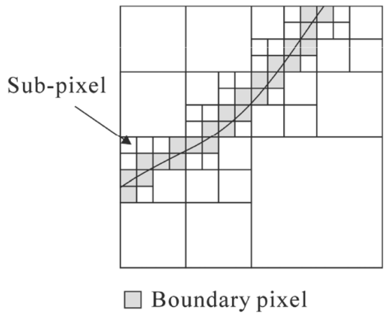
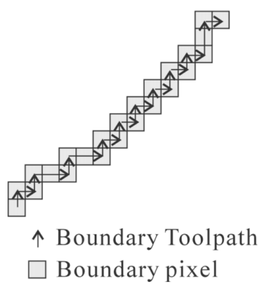


Figure 14 The boundary toolpath of the pixel (1, 3) in Figure 11



nozzle diameter for fused deposition modeling (FDM) process) will be filtered out or their sizes are approximated as equal to the tool size. So in the proposed methodology, it is reasonable to set the resolution of the slab grid equal to the tool resolution of the LM machine being used for making the prototype.

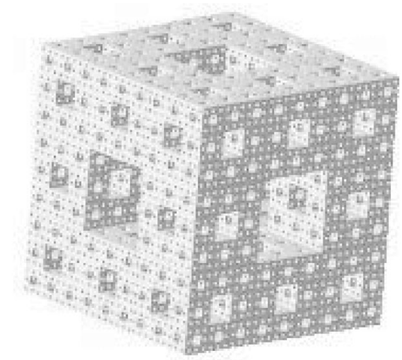
3.3 Refining resolution

To refine the boundary, each Part-IN pixel must be further sub-divided so that a smoother boundary can be resulted. However, the refining resolution must also be determined so that the refining process can be stopped once the boundary is approximated in an acceptable accuracy. In this case, there are two factors that must be considered. They are the required quality of the prototype and the resolution of the LM machine used for making the prototype. For the former factor, it is highly related to the resolution of the fractal object while for the latter factor, it is used to determine how fine a prototype can be made by the machine. In general, it is meaningless to have a higher resolution of the fractal object than the LM machine resolution. In this methodology, the resolution of the quadtree can be set according to the machine resolution.

4. Results

To illustrate the proposed methodology, two examples have been given. In the first example, a Menger Sponge (Figure 15) which is an IFS fractal model was fabricated by the FDM process. Applying the proposed methodology, slab grid is generated and 2D slab model was obtained in each layer, as shown in Figure 16. In this simple example, no Part-IN pixels are found. The boundary pixels are identified and the boundary of the slab model is traced. The toolpath information is generated and a physical prototype was fabricated (Figure 17).

Figure 15 Menger Sponge



In the second example, a QJ-set was used as an example. Similarly, slab model in each layer must be obtained. In Figure 18, an example slab model is shown. Based on the slab models, the boundary in each layer is extracted and the toolpath for the whole fractal object is generated accordingly.

5. Discussion

In this paper, a methodology is proposed for generating the toolpath information of a fractal object from its mathematical model for the LM technology. However, there are some drawbacks that must be considered when this methodology is applied. Firstly, there would be a large number of pixels in each slab model and the computational effort for checking the interior property of each pixel would be large. Moreover, the amount of memory required for storing the slab models is also needed to be considered. Secondly, each Part-IN pixel has to be refined and similar problems are induced.

The refining resolution of the methodology is set according to the machine resolution. As mentioned above (such as that shown in Figure 4), a fractal is infinitely complex when it is magnified and it is impossible to obtain the exact shape of the fractal unless the iteration process for generating the fractal is terminated in a predefined level (or an approximation level). As a result, it is reasonable to use the machine resolution as the refining resolution as it infers the finest details that can be made by the machine. The problem of the accuracy of the prototype as compared to the model is now become the problem of how fine a fractal prototype a user would like to have.

After the 2D pixels in each layer of a fractal are obtained, an approximation model can be constructed by extruding each 2D pixel into a 3D voxel. For example, in Figure 19, the 2D pixels of a QJ-set are extruded into a number of voxels. It provides another possible method for visualizing the fractal object.

The proposed methodology can be applied to IFS or complex fractal. However, some of the fractals are constructed from more than one kind of fractal objects, such as multi-IFS fractals (e.g. human body). In this case, the methodology may need to be modified in order to handle this type of fractal.

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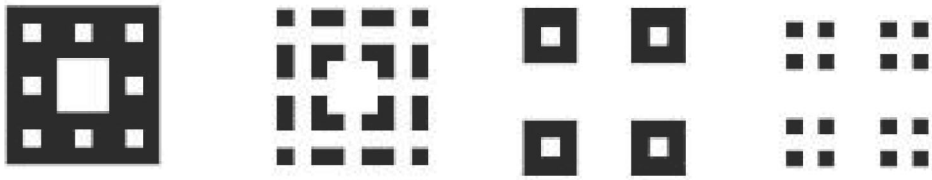
Figure 16 Slab models of Menger Sponge in some layers



(a) (b) (c) (d)

Figure 17 Physical prototype of a Menger Sponge

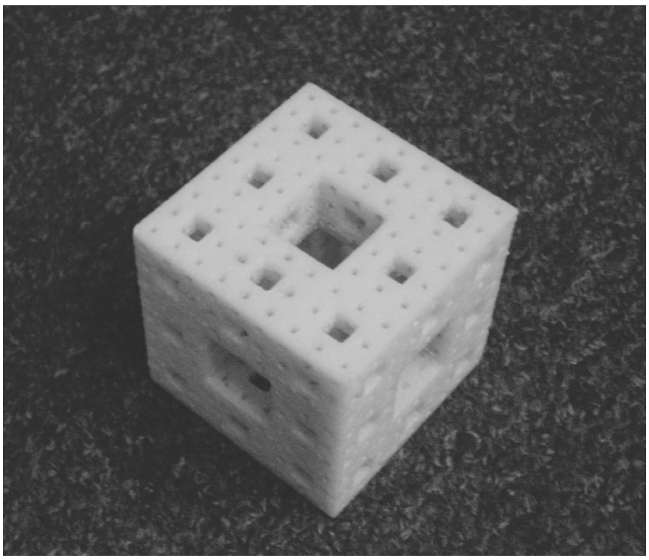
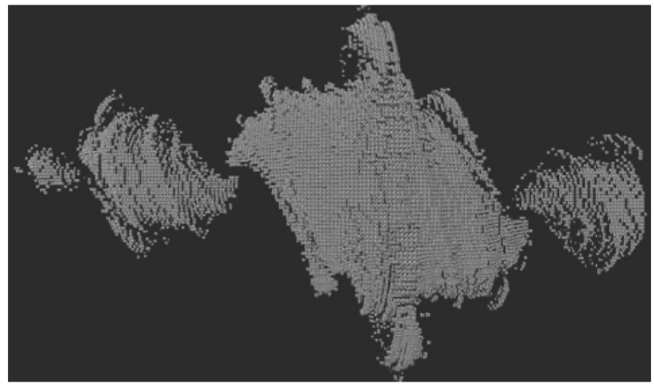


Figure 18 Slab model of a QJ-set

6. Conclusion

Applying the proposed methodology, a fractal object can be fabricated by a LM process without the need to represent the object by the euclidean geometry in a CAD system. This is an advantage as investigating a proper representation scheme for the fractal geometry is still an open issue in CAD. Moreover, once the mathematical model of the fractal object is defined, its toolpath information can be extracted by the proposed methodology no matter the object is an IFS or a complex fractal. As a result, the fabrication of a fractal prototype becomes possible.

Figure 19 A QJ-set fractal model displayed as a number of slab models



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