# Fractal properties of the liver for 3D liver surgery planning – computation beyond Euclidian geometry

M. T. J. Mohr[1], A. G. Schreyer[2]

[1] BioMedical Data Processing Group (BMDPG), Regensburg, Germany (Director: M. T. J. Mohr, M. D.)

[2] Department of Radiology, University of Regensburg, Regensburg, Germany (Director: Prof. St. Feuerbach, M. D.)

Successful liver surgery depends not only on excellent technological infrastructure, a well-adapted operating team, and long years of surgical visceral experience, but also profound anatomical knowledge. The individual liver anatomy is characterized by an agglomeration of self-similar three-dimensional (3D) tissue units. However, these units cannot be measured by simple Euclidian geometry, but must be subjected to the so-called fractal dimension which depicts a measure for the self-similarity and self-affinity of an irregular object.

Mathematical topology defines the various terms of dimension. The Euclidian dimension (ED) is characterized by mathematically exactly describable objects (dot, line, square, circle, sphere) with ideal side border properties. Thus, the ED always results in finite and unbroken dimensions (1.0, 2.0, and 3.0). The fractal dimension (FD), however, describes objects with non-ideal side properties, such as leaves, country and mountain edges, and biological organs. Therefore, the FD delivers broken dimension values (0.x, 1.x, and 2.x). The extreme values for FD 1.0, 2.0, and 3.0, hence, are identical to the ED.

Thus, biological organs with their individual properties can only be described with the FD. One method to use is the so-called BESICOVICH-HAUSSDORFF dimension (BHD) which depicts a geometrical relation of self-affine geometrical aspects of an arbitrary object to be repeated in a smaller or enlarged size of the same relation. In other words, since many biological objects, such as leaves, are self-affine, a measure for self-affinity is computed against the two- (2D) or 3D aspect of this object which results in this object’s FD.

The liver as surgical organ possesses a FD between 2.0x and 2.9x. Due to the curved form of its surface and inner volume, the measurement of the liver’s FD is not trivial at all, and auxiliary procedures such as the computation of FDs of individual compartments are often needed. Apart from the mathematical computation of the liver’s FD, much more interesting for the liver surgeon is the planning of his surgery which most often implies a volume reduction of one third or two thirds. Anatomical boundaries such as arterial and venous vessels have to be observed in order not to destroy the remaining liver’s vasculature and nutrition. The determination of the fractal properties (FP) of the liver segment to be resected, therefore, can greatly help the surgeon to find his way away from the “noli me tangere” regions which have to be spared. Furthermore, using this geometrical technique together with computational procedures, 3D models of the liver and thus surgical planning can be prepared on the computer.

This presentation not only explains how to determine the FD and the FP of the liver, but shows practical examples of how to use this procedure for surgical resection of specific liver segments. Furthermore, software specimens are presented which enable a 2D and 3D fractal reconstruction of the liver segments of interest, in addition to liver volume rendering.