The Mandelbulb: first 'true' 3D image of famous fractal

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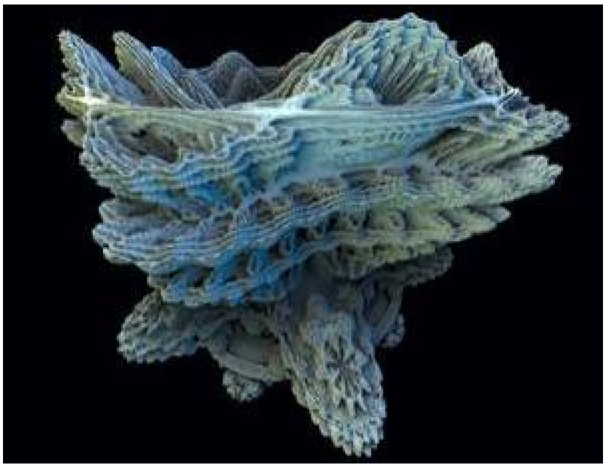

Reaching new dimensions (Image: Daniel White) **See:** More images

It may look like a piece of virtuoso knitting, but the makers of an image they call the Mandelbulb (see right) claim it is most accurate three-dimensional representation to date of the most famous fractal equation: the Mandelbrot set.

Fractal figures are generated by an "iterative" procedure: you apply an equation to a number, apply the same equation to the result and repeat that process over and over again. When the results are translated into a geometric shape, they can produce striking "self-similar" images, forms that contain the same shapes at different scales; for instance, some look uncannily like a snowflake. The tricky part is finding an equation that produces an interesting image.

The most famous fractal equation is the 2D Mandelbrot set, named after the mathematician Benoît Mandelbrot of Yale University, who coined the name "fractals" for the resulting shapes in 1975.

But there are many other types of fractal, both in two and three dimensions. The "Menger sponge" is one of the simplest 3D examples.

Fake fractal

There have been previous attempts at a 3D Mandelbrot image, but they do not display real fractal behaviour, says Daniel White, an amateur fractal image maker based in Bedford, UK.

Spinning the 2D Mandelbrot fractal like wood on a lathe, raising and lowering certain points, or invoking higher-dimensional mathematics can all produce apparently three-dimensional Mandelbrots. Yet none of these techniques offer the detail and self-similar shapes that White believes represent a true 3D fractal image.

Two years ago, he decided to find a "true" 3D version of the Mandelbrot.

The next dimension

"I was trying to see how the original 2D Mandelbrot worked and translate that to the third dimension," he explains. "You can use complex maths but you can also look at things geometrically."

This approach works thanks to the properties of the "complex plane", a mathematical landscape where ordinary numbers run from "east" to "west", while "imaginary" numbers, based on the square root of -1, run from "south" to "north". Multiplying numbers on the complex plane is the same as rotating it, and addition is like shifting the plane in a particular direction.

To create the Mandelbrot set, you just repeat these geometrical actions for every point in the plane. Some will balloon to infinity, escaping the set entirely, while others shrink down to zero. The different colours on a typical image reflect the number of iterations before each point hits zero.

White wondered if performing these same rotations and shifts in a 3D space would capture the essence of the Mandelbrot set without using complex numbers – real numbers plus imaginary numbers – which do not apply in three dimensions because they are on only two axes. In November 2007, White published a formula for a shape that came pretty close.

Higher power

The formula published by White gave good results, but still lacked true fractal detail. Collaborating with the members of Fractal Forums, a website for fractal admirers, he continued his search. It was another member, Paul Nylander, who eventually realised that raising White's formula to a higher power – equivalent to increasing the number of rotations – would produce what they were looking for.

White's search isn't over, though. He admits the Mandelbulb is not quite the "real" 3D Mandelbrot. "There are still 'whipped cream' sections, where there isn't detail," he explains. "If the real thing does exist – and I'm not saying 100 per cent that it does – one would expect even more variety than we are currently seeing."

Part of the problem is that extending the Mandelbrot set to 3D requires many subjective choices that influence the outcome. For example, you could extend a flat plane to 3D by stretching it to form a box, but you could also turn it into a sphere.

"It's an interesting academic exercise to think what you should get," says Martin Turner, a computer scientist specialising in fractal images at the University of Manchester, UK, "but it all depends on what properties you want to keep in the third dimension."

The equations White used may get the job done, but the system of algebra used is not applicable to all 3D mathematics. "The next stage is mathematical rigour," says Turner.

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